The binomial expansion 8A

1 a $(x + y)^3$
The $(n + 1)$th row of Pascal's triangle gives the coefficients in the expansion of $(a + b)^n$.
$3 + 1 = 4$th row

b $(3x - 7)^{15}$
$15 + 1 = 16$th row

c $\left(2x + \frac{1}{2}\right)^n$
$n + 1 = (n + 1)$th row

d $(y - 2x)^{n+4}$
$n + 4 + 1 = (n + 5)$th row

2 a $(x + y)^4$ has coefficients and terms

1	4	6	4	1
x^4	x^3y	x^2y^2	xy^3	y^4

$$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$
$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

b $(p + q)^5$ has coefficients and terms

1	5	10	10	5	1
p^5	p^4q	p^3q^2	p^2q^3	pq^4	q^5

$$(p + q)^5 = 1p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + 1q^5$$
$$= p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$$

c $(a - b)^3$ has coefficients and terms

1	3	3	1
a^3	$a^2(-b)$	$a(-b)^2$	$(-b)^3$

$$(a - b)^3 = 1a^3 - 3a^2b + 3ab^2 - 1b^3$$
$$= a^3 - 3a^2b + 3ab^2 - b^3$$

d $(x + 4)^3$ has coefficients and terms

1	3	3	1
x^3	$x^2 4$	$x4^2$	4^3

$$(x + 4)^3 = 1x^3 + 3x^2(4) + 3x(4)^2 + 4^3$$
$$= x^3 + 12x^2 + 48x + 64$$

e $(2x - 3)^4$ has coefficients and terms

1	4	6	4	1
$(2x)^4$	$(2x)^3(-3)$	$(2x)^2(-3)^2$	$(2x)(-3)^3$	$(-3)^4$

$$(2x - 3)^4 = 1(2x)^4 + 4(2x)^3(-3) + 6(2x)^2(-3)^2 + 4(2x)(-3)^3 + 1(-3)^4$$
$$= 16x^4 - 96x^3 + 216x^2 - 216x + 81$$

2 f $(a + 2)^5$ has coefficients and terms

1	5	10	10	5	1
a^2	$a^4 2$	$a^3 2^2$	$a^2 2^3$	$a 2^4$	2^5

$$(a + 2)^5 = 1a^5 + 5a^4(2) + 10a^3(2)^2 + 10a^2(2)^3 + 5a(2)^4 + 32$$
$$= a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32$$

g $(3x - 4)^4$ has coefficients and terms

1	4	6	4	1
$(3x)^4$	$(3x)^3(-4)$	$(3x)^2(-4)^2$	$(3x)(-4)^3$	$(-4)^4$

$$(3x - 4)^4 = 1(3x)^4 + 4(3x)^3(-4) + 6(3x)^2(-4)^2 + 4(3x)(-4)^3 + 1(-4)^4$$
$$= 81x^4 - 432x^3 + 864x^2 - 768x + 256$$

h $(2x - 3y)^4$ has coefficients and terms

1	4	6	4	1
$(2x)^4$	$(2x)^3(-3y)$	$(2x)^2(-3y)^2$	$(2x)(-3y)^3$	$(-3y)^4$

$$(2x - 3y)^4 = 1(2x)^4 + 4(2x)^3(-3y) + 6(2x)^2(-3y)^2 + 4(2x)(-3y)^3 + 1(-3y)^4$$
$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

3 a $(4 + x)^4$ has coefficients 1 4 6 **4** 1
The bold number is the coefficient of the term $4x^3$.
Term is $4 \times 4x^3 = 16x^3$.
Coefficient = 16

b $(1 - x)^5$ has coefficients 1 5 10 **10** 5 1
The bold number is the coefficient of the term $1^2(-x)^3$.
Term is $10 \times 1^2(-x)^3 = -10x^3$.
Coefficient = −10

c $(3 + 2x)^3$ has coefficients 1 3 3 **1**
The bold number is the coefficient of the term $(2x)^3$.
Term is $1 \times (2x)^3 = 8x^3$.
Coefficient = 8

d $(4 + 2x)^5$ has coefficients 1 5 10 **10** 5 1
The bold number is the coefficient of the term $4^2(2x)^3$.
Term is $10 \times 4^2(2x)^3 = 1280x^3$.
Coefficient = 1280

e $(2 + x)^6$ has coefficients 1 6 15 **20** 15 6 1
The bold number is the coefficient of the term 2^3x^3.
Term is $20 \times 2^3x^3 = 160x^3$.
Coefficient = 160

f $\left(4 - \frac{1}{2}x\right)^4$ has coefficients 1 4 6 **4** 1

The bold number is the coefficient of the term $4\left(-\frac{1}{2}x\right)^3$

3 f Term is $4 \times 4\left(-\frac{1}{2}x\right)^3 = -2x^3$

Coefficient = −2

g $(x + 2)^5$ has coefficients 1 5 **10** 10 5 1
The bold number is the coefficient of the term $x^3 2^2$.
Term is $10 \times x^3 2^2 = 40x^3$.
Coefficient = 40

h $(3 - 2x)^4$ has coefficients 1 4 6 **4** 1
The bold number is the coefficient of the term $3(-2x)^3$.
Term is $4 \times 3(-2x)^3 = -96x^3$.
Coefficient = −96

4 $(1 + 2x)^3$ has coefficients and terms

1	3	3	1
1^3	$1^2(2x)$	$1(2x)^2$	$(2x)^3$

Hence $(1 + 2x)^3 = 1 + 6x + 12x^2 + 8x^3$

$(1 + 3x)(1 + 2x)^3$
$= (1 + 3x)(1 + 6x + 12x^2 + 8x^3)$
$= 1(1 + 6x + 12x^2 + 8x^3) + 3x(1 + 6x + 12x^2 + 8x^3)$
$= 1 + 6x + 12x^2 + 8x^3 + 3x + 18x^2 + 36x^3 + 24x^4$
$= 1 + 9x + 30x^2 + 44x^3 + 24x^4$

5 $(2 + y)^3$ has coefficients and terms

1	3	3	1
2^3	2^2y	$2y^2$	y^3

Hence $(2 + y)^3 = 8 + 12y + 6y^2 + y^3$

Substitute $y = x - x^2$
$(2 + x - x^2)^3 = 8 + 12(x - x^2) + 6(x - x^2)^2 + (x - x^2)^3$
$= 8 + 12x(1 - x) + 6x^2(1 - x^2)^2 + x^3(1 - x)^3$

Now
$(1 - x)^3 = (1 - x)(1 - x)^2$
$(1 - x)^3 = (1 - x)(1 - 2x + x^2)$
$(1 - x)^3 = 1 - 2x + x^2 - x + 2x^2 - x^3$
$(1 - x)^3 = 1 - 3x + 3x^2 - x^3$

Or, using Pascal's triangle
$(1 - x)^3 = 1(1)^3 + 3(1)^2(-x) + 3(1)(-x)^2 + 1(-x)^3$
$(1 - x)^3 = 1 - 3x + 3x^2 - x^3$

So $(2 + x - x^2) = 8 + 12x(1 - x) + 6x^2(1 - 2x + x^2) + x^3(1 - 3x + 3x^2 - x^3)$
$= 8 + 12x - 12x^2 + 6x^2 - 12x^3 + 6x^4 + x^3 - 3x^4 + 3x^5 - x^6$
$= 8 + 12x - 6x^2 - 11x^3 + 3x^4 + 3x^5 - x^6$

6 $(2 + ax)^3$ has coefficients 1 3 **3** 1
The bold number is the coefficient of the term $2(ax)^2$
Term in x^2 is $3 \times 2(ax)^2 = 6a^2x^2$
Coefficient of x^2 is $6a^2$

So $6a^2 = 54$
$a^2 = 9$
$a = \pm 3$

7 $(3 + bx)^3$ has coefficients and terms

1	3	3	1
3^3	$3^2(bx)$	$3(bx)^2$	$(bx)^3$

$(3 + bx)^3 = 1 \times 3^3 + 3 \times 3^2bx + 3 \times 3(bx)^2 + 1 \times (bx)^3$
$= 27 + 27bx + 9b^2x^2 + b^3x^3$

So $(2 - x)(3 + bx)^3 = (2 - x)(27 + 27bx + 9b^2x^2 + b^3x^3)$

Term in x^2 is $2 \times 9b^2x^2 - x \times 27bx = 18b^2x^2 - 27bx^2$
Coefficient of x^2 is $18b^2 - 27b$

So $18b^2 - 27b = 45$
$2b^2 - 3b = 5$
$2b^2 - 3b - 5 = 0$
$(2b - 5)(b + 1) = 0$
$b = \frac{5}{2}, -1$

8 The coefficients are 1, 3, **3**, 1
The term in x^2 is $3(p)(-2x)^2 = 12px^2$
So the coefficient of the term x^2 is $12p$

9 $500\left(1+\frac{X}{100}\right)^5$
The coefficients are 1, 5, 10, 10, 5, 1

$$500\left(1+\frac{X}{100}\right)^5 = 500\left(1(1)^5+5(1)^4\left(\frac{X}{100}\right)+10(1)^3\left(\frac{X}{100}\right)^2+...\right)$$
$$= 500\left(1+\frac{X}{20}+\frac{X^2}{1000}+...\right)$$
$$\approx 500 + 25X + \frac{X^2}{2}$$
$A = 500$, $B = 25$, $C = \frac{1}{2}$

Challenge

$(x^2 - \frac{1}{2x})^3$
Coefficients are 1 3 **3** 1
Third term is $3(x^2)^1(-\frac{1}{2x})^2 = \frac{3x^2}{4x^2} = \frac{3}{4}$