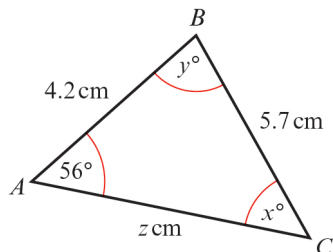


Trigonometric ratios 9E

1 a



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin x^\circ}{4.2} = \frac{\sin 56^\circ}{5.7}$$

$$\sin x^\circ = \frac{4.2 \sin 56^\circ}{5.7}$$

$$x^\circ = \sin^{-1}\left(\frac{4.2 \sin 56^\circ}{5.7}\right)$$

$$= 37.65\dots^\circ$$

$$x = 37.7 (3 \text{ s.f.})$$

So $y^\circ = 180^\circ - (56^\circ + 37.7^\circ)$

$$= 86.3^\circ$$

$$y = 86.3 (3 \text{ s.f.})$$

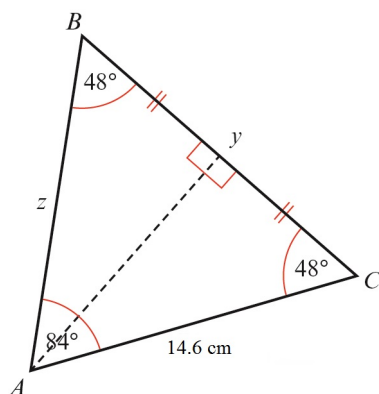
Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{z}{\sin y^\circ} = \frac{5.7}{\sin 56^\circ}$$

So $z = \frac{5.7 \sin y^\circ}{\sin 56^\circ}$

$$= 6.86 (3 \text{ s.f.})$$

b $x^\circ = 180^\circ - (48 + 84)^\circ$
 $x^\circ = 48^\circ$



As angle B = angle C, $z = 14.6 \text{ cm}$.

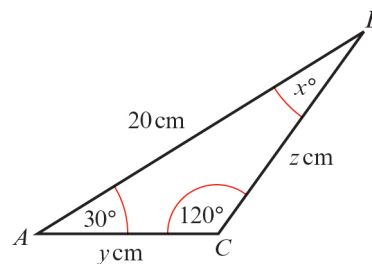
b Using the line of symmetry through A

$$\cos 48^\circ = \frac{\frac{y}{2}}{14.6}$$

So $y = 29.2 \cos 48^\circ$

$$= 19.5 (3 \text{ s.f.})$$

c



$$x^\circ = 180^\circ - (120^\circ + 30^\circ)$$

$$= 30^\circ$$

Using the line of symmetry through C

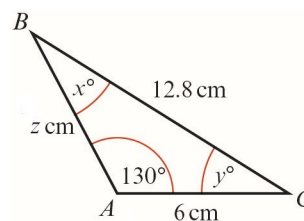
$$\cos 30^\circ = \frac{10}{y}$$

So $y = \frac{10}{\cos 30^\circ}$

$$= 11.5 (3 \text{ s.f.})$$

Since $\triangle ABC$ is isosceles with $AC = CB$
 $z = 11.5 (3 \text{ s.f.})$.

d



Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\frac{\sin 130^\circ}{12.8} = \frac{\sin x^\circ}{6}$$

So $\sin x^\circ = \frac{6 \sin 130^\circ}{12.8}$

$$= 0.35908\dots$$

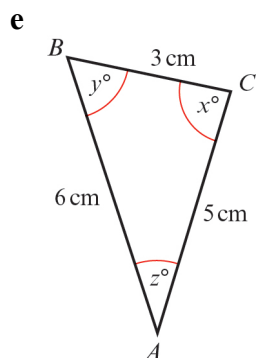
$$\Rightarrow x = 21.0 (3 \text{ s.f.})$$

So $y^\circ = 180^\circ - (130^\circ + x^\circ)$

$$= 28.956\dots^\circ$$

1 d $\Rightarrow y = 29.0$ (3 s.f.)

$$\begin{aligned} \text{Using } \frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{z}{\sin y^\circ} &= \frac{12.8}{\sin 130^\circ} \\ \text{So } z &= \frac{12.8 \sin y^\circ}{\sin 130^\circ} \\ &= 8.09 \text{ (3 s.f.)} \end{aligned}$$

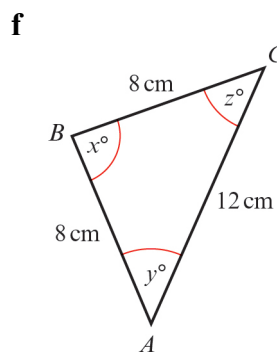


$$\begin{aligned} \text{Using } \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ \cos x^\circ &= \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5} \\ &= -0.0\dot{6} \\ x &= 93.8 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{Using } \frac{\sin B}{b} &= \frac{\sin C}{c} \\ \frac{\sin y^\circ}{5} &= \frac{\sin x^\circ}{6} \\ \sin y^\circ &= \frac{5 \sin x^\circ}{6} \\ y^\circ &= \sin^{-1} \left(\frac{5 \sin x^\circ}{6} \right) \\ &= 56.25\dots^\circ \\ y &= 56.3 \text{ (3 s.f.)} \end{aligned}$$

Using the angle sum for a triangle

$$\begin{aligned} z^\circ &= 180^\circ - (x + y)^\circ \\ &= 29.926\dots^\circ \\ z &= 29.9 \text{ (3 s.f.)} \end{aligned}$$



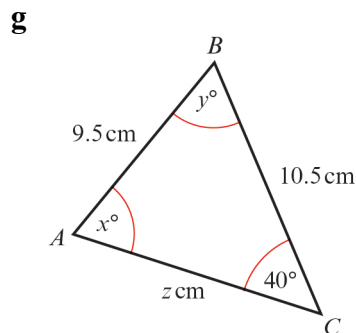
Using the line of symmetry through B

$$\begin{aligned} \cos y^\circ &= \frac{6}{8} \\ &= \frac{3}{4} \\ y^\circ &= \cos^{-1} \left(\frac{3}{4} \right) \\ &= 41.40\dots \\ y &= 41.4 \text{ (3 s.f.)} \end{aligned}$$

As the triangle is isosceles

$$\begin{aligned} z &= y \\ &= 41.4 \text{ (3 s.f.)} \end{aligned}$$

$$\begin{aligned} \text{So } x^\circ &= 180^\circ - (y + z)^\circ \\ &= 97.2^\circ \\ x &= 97.2 \text{ (3 s.f.)} \end{aligned}$$



$$\begin{aligned} \text{Using } \frac{\sin A}{a} &= \frac{\sin C}{c} \\ \frac{\sin x^\circ}{10.5} &= \frac{\sin 40^\circ}{9.5} \\ \sin x^\circ &= \frac{10.5 \sin 40^\circ}{9.5} \\ x^\circ &= \sin^{-1} \left(\frac{10.5 \sin 40^\circ}{9.5} \right) \text{ or} \\ x^\circ &= 180^\circ - \sin^{-1} \left(\frac{10.5 \sin 40^\circ}{9.5} \right) \\ x^\circ &= 45.27^\circ \text{ or } x^\circ = 134.728\dots^\circ \\ x &= 45.3 \text{ (3 s.f.) or } x = 135 \text{ (3 s.f.)} \end{aligned}$$

1 g Using $\frac{b}{\sin B} = \frac{c}{\sin C}$
 $\frac{z}{\sin y^\circ} = \frac{9.5}{\sin 40^\circ}$
 $z = \frac{9.5 \sin y^\circ}{\sin 40^\circ}$

When $x = 45.3$

$$y^\circ = 180^\circ - (40 + 45.3)^\circ$$

$$= 94.7^\circ$$

So $y = 94.7$ (3 s.f.)

$$z = \frac{9.5 \sin y^\circ}{\sin 40^\circ}$$

$$= 14.7 \text{ (3 s.f.)}$$

When $x = 134.728\dots$

$$y^\circ = 180^\circ - (40 + 134.72\dots)^\circ$$

$$= 5.27^\circ$$

So $y = 5.27$ (3 s.f.)

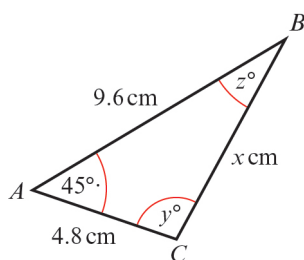
$$z = \frac{9.5 \sin y^\circ}{\sin 40^\circ}$$

$$= 1.36 \text{ (3 s.f.)}$$

So $x = 45.3$, $y = 94.7$, $z = 14.7$

or $x = 135$, $y = 5.27$, $z = 1.36$

h



Using $a^2 = b^2 + c^2 - 2bc \cos A$

$$x^2 = 4.8^2 + 9.6^2 - 2 \times 4.8 \times 9.6 \times \cos 45^\circ$$

$$= 50.03\dots$$

$$x = 7.07 \text{ (3 s.f.)}$$

Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin y^\circ}{9.6} = \frac{\sin 45^\circ}{x}$$

$$\sin y^\circ = \frac{9.6 \sin 45^\circ}{x}$$

$$y^\circ = \sin^{-1} \left(\frac{9.6 \sin 45^\circ}{x} \right)$$

h $y^\circ = 180 - 73.68\dots^\circ$ (y cannot be acute)

$$y^\circ = 106.32\dots^\circ$$

$$y = 106 \text{ (3 s.f.)}$$

Then

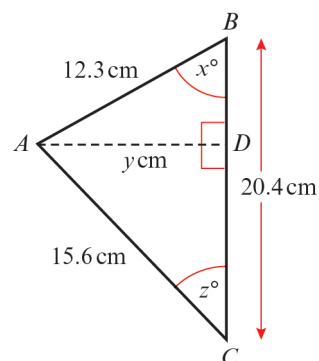
$$z^\circ = 180^\circ - (45 + 106.32\dots)^\circ$$

$$= 28.68\dots^\circ$$

$$z = 28.7 \text{ (3 s.f.)}$$

So $x = 7.07$, $y = 107$, $z = 28.7$

i



Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\cos x^\circ = \frac{20.4^2 + 12.3^2 - 15.6^2}{2 \times 20.4 \times 12.3}$$

$$= 0.6458\dots$$

$$x = 49.77\dots^\circ$$

$$x = 49.8 \text{ (3 s.f.)}$$

In right-angled triangle ABD

$$\sin x^\circ = \frac{y}{12.3}$$

$$\text{So } y = 12.3 \sin x^\circ$$

$$= 9.39 \text{ (3 s.f.)}$$

In right-angled triangle ACD

$$\sin z^\circ = \frac{y}{15.6}$$

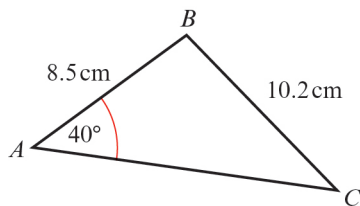
$$= 0.60199\dots$$

$$z^\circ = 37.01\dots^\circ$$

$$z = 37.0 \text{ (3 s.f.)}$$

So $x = 49.8$, $y = 9.39$, $z = 37.0$

2 a



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin C}{8.5} = \frac{\sin 40^\circ}{10.2}$$

$$\sin C = \frac{8.5 \sin 40^\circ}{10.2}$$

$$C = \sin^{-1}\left(\frac{8.5 \sin 40^\circ}{10.2}\right)$$

$$= 32.388\dots^\circ$$

$$= 32.4^\circ \text{ (3 s.f.)}$$

$$B = 180^\circ - (40 + C)^\circ$$

$$= 107.6\dots^\circ$$

$$B = 108^\circ \text{ (3 s.f.)}$$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$b = \frac{10.2 \sin B}{\sin 40^\circ}$$

$$= 15.1 \text{ cm (3 s.f.)}$$

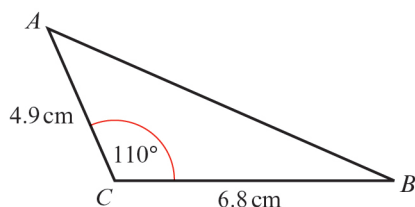
$$\text{Area} = \frac{1}{2}ac \sin B$$

$$= \frac{1}{2} \times 10.2 \times 8.5 \times \sin 108^\circ$$

$$= 41.228$$

$$= 41.2 \text{ cm}^2 \text{ (3 s.f.)}$$

b



Using $c^2 = a^2 + b^2 - 2ab \cos C$

$$AB^2 = 6.8^2 + 4.9^2 - 2 \times 6.8 \times 4.9 \times \cos 110^\circ$$

$$= 93.04\dots$$

$$AB = 9.6458\dots$$

$$= 9.65 \text{ cm (3 s.f.)}$$

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

2 b $\sin A = \frac{6.8 \sin 110^\circ}{AB}$

$$= 0.66245\dots$$

$$A = 41.49^\circ$$

$$= 41.5^\circ \text{ (3 s.f.)}$$

$$\text{So } B = 180^\circ - (110 + A)^\circ$$

$$= 28.5^\circ \text{ (3 s.f.)}$$

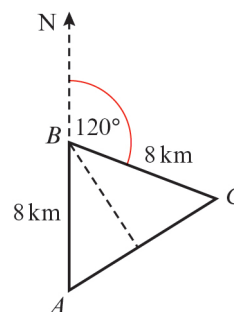
$$\text{Area} = \frac{1}{2}ac \sin B$$

$$= \frac{1}{2} \times 6.8 \times 4.9 \times \sin 110^\circ$$

$$= 15.655\dots$$

$$= 15.7 \text{ cm}^2 \text{ (3 s.f.)}$$

3



a Angle $ABC = 180^\circ - 120^\circ = 60^\circ$

As $\angle A = \angle C$, all angles are 60° .

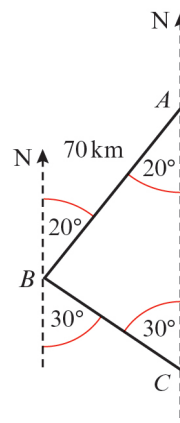
It is an equilateral triangle.

So $AC = 8 \text{ km}$.

b As $\angle BAC = 60^\circ$,

the bearing of C from A is 060° .

4



From the diagram

$$\angle ABC = 180^\circ - (20 + 30)^\circ$$

$$= 130^\circ$$

4 Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

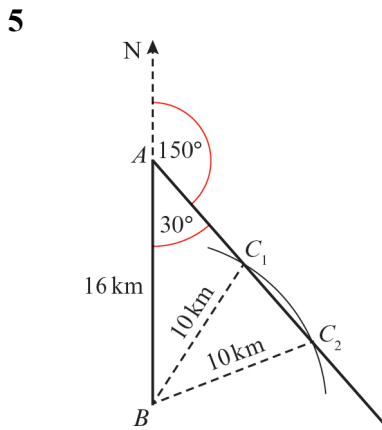
$$\frac{AC}{\sin 130^\circ} = \frac{70}{\sin 30^\circ}$$

$$AC = \frac{70 \sin 130^\circ}{\sin 30^\circ}$$

$$= 107.246\dots$$

$AC = 107 \text{ km (3 s.f.)}$

From the diagram, the bearing of C from A is 180° .



Using the sine rule

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{16} = \frac{\sin 30^\circ}{10}$$

$$\sin C = \frac{16 \sin 30^\circ}{10}$$

$$= 0.8$$

$C = \sin^{-1}(0.8)$ or $C = 180^\circ - \sin^{-1}(0.8)$

$C = 53.1^\circ$ or $C = 126.9^\circ$

$\angle AC_2B = 53.1^\circ, \angle AC_1B = 127^\circ$ (3 s.f.)

(Store the correct values; these are not required answers.)

Triangle BC_1C_2 is isosceles, so C_1C_2 can be found using this triangle, without finding AC_1 and AC_2 .

Use the line of symmetry through B

$$\cos \angle C_1C_2B = \frac{\frac{1}{2}C_1C_2}{10}$$

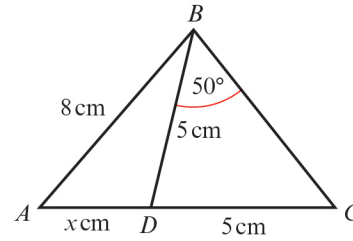
$$\Rightarrow C_1C_2 = 20 \cos \angle C_1C_2B$$

$$= 20 \cos \angle AC_2B$$

$$= 20 \cos 53.1^\circ$$

$$\Rightarrow C_1C_2 = 12 \text{ km}$$

6 a



In the isosceles $\triangle BDC$

$$\angle BDC = 180^\circ - (50 + 50)^\circ$$

$$= 80^\circ$$

So $\angle BDA = 180^\circ - 80^\circ$

$$= 100^\circ$$

Using the sine rule in $\triangle ABD$

$$\frac{\sin A}{a} = \frac{\sin D}{d}$$

$$\Rightarrow \frac{\sin A}{5} = \frac{\sin 100^\circ}{8}$$

$$\Rightarrow \sin A = \frac{5 \sin 100^\circ}{8}$$

So $A = \sin^{-1}\left(\frac{5 \sin 100^\circ}{8}\right)$

$$= 37.9886\dots$$

$$\angle ABD = 180^\circ - (100 + A)^\circ$$

$$= 42.01\dots^\circ$$

Using $\frac{b}{\sin B} = \frac{d}{\sin D}$

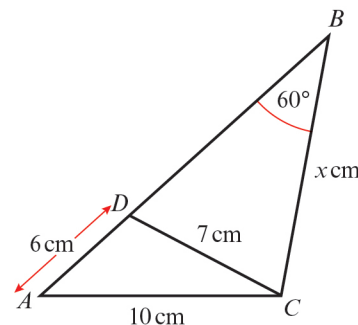
$$\frac{x}{\sin B} = \frac{8}{\sin 100^\circ}$$

$$x = \frac{8 \sin B}{\sin 100^\circ}$$

$$= 5.436\dots$$

$x = 5.44$ (3 s.f.)

b



In $\triangle ADC$, using $\cos A = \frac{c^2 + d^2 - a^2}{2cd}$

$$6 \text{ b } \cos A = \frac{6^2 + 10^2 - 7^2}{2 \times 6 \times 10}$$

$$= 0.725$$

$$\text{So } A = 43.53\dots^\circ$$

Using the sine rule in $\triangle ABC$

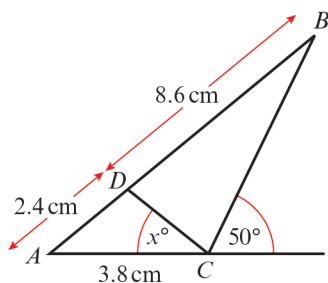
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\text{So } \frac{x}{\sin A} = \frac{10}{\sin 60^\circ}$$

$$\Rightarrow x = \frac{10 \sin A}{\sin 60^\circ}$$

$$\text{So } x = 7.95 \text{ (3 s.f.)}$$

c



In $\triangle ABC$, $c = 11 \text{ cm}$, $b = 3.8 \text{ cm}$,
 $\angle ACB = 130^\circ$, $(180^\circ - 50^\circ)$

$$\text{Using } \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\sin B = \frac{3.8 \sin 130^\circ}{11}$$

$$= 0.2646\dots$$

$$B = 15.345\dots^\circ$$

$$\text{So } A = 180^\circ - (130 + B)^\circ$$

$$= 34.654\dots^\circ$$

In $\triangle ADC$, $c = 2.4 \text{ cm}$, $d = 3.8 \text{ cm}$,

$$A = 34.654\dots^\circ$$

Using the cosine rule:

$$a^2 = c^2 + d^2 - 2cd \cos A$$

$$\text{So } DC^2 = 2.4^2 + 3.8^2 - 2 \times 2.4 \times 3.8 \times \cos A$$

$$= 5.1959\dots$$

$$\Rightarrow DC = 2.279\dots \text{ cm}$$

Using the sine rule:

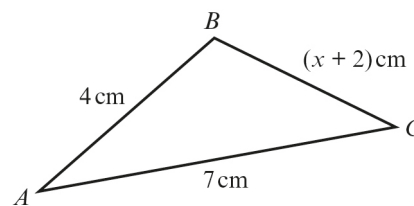
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin x^\circ = \frac{2.4 \sin A}{DC}$$

$$= 0.59869\dots$$

$$x = 36.8 \text{ (3 s.f.)}$$

7



a As $AB + BC > AC$

$$4 + (x + 2) > 7$$

$$\Rightarrow x + 2 > 3$$

$$\Rightarrow x > 1$$

As $AB + AC > BC$

$$4 + 7 > x + 2$$

$$\Rightarrow 9 > x$$

$$\text{So } 1 < x < 9$$

b Using $b^2 = a^2 + c^2 - 2ac \cos B$

$$\text{i } 7^2 = (x + 2)^2 + 4^2 - 2(x + 2) \times 4 \times \cos 60^\circ$$

$$49 = x^2 + 4x + 4 + 16 - 4(x + 2)$$

$$49 = x^2 + 4x + 4 + 16 - 4x - 8$$

$$\text{So } x^2 = 37$$

$$\Rightarrow x = 6.08 \text{ (3 s.f.)}$$

$$\text{Area} = \frac{1}{2} ac \sin B$$

$$= \frac{1}{2} \times 8.08 \times 4 \times \sin 60^\circ$$

$$= 13.9949\dots$$

$$= 14.0 \text{ cm}^2 \text{ (3 s.f.)}$$

$$\text{ii } 7^2 = (x + 2)^2 + 4^2$$

$$- 2 \times (x + 2) \times 4 \times \cos 45^\circ$$

$$49 = x^2 + 4x + 4 + 16$$

$$- (8 \cos 45^\circ)x - 16 \cos 45^\circ$$

So:

$$x^2 + (4 - 8 \cos 45^\circ)x$$

$$- (29 + 16 \cos 45^\circ) = 0$$

$$\text{or } x^2 + 4(1 - \sqrt{2})x$$

$$- (29 + 8\sqrt{2}) = 0$$

Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ with } a = 1$$

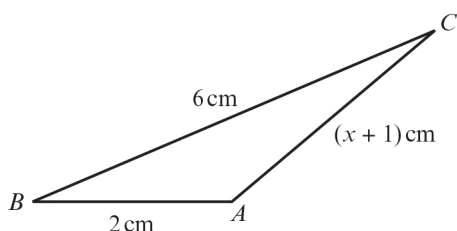
$$b = 4 - 8 \cos 45^\circ$$

$$= 4(1 - \sqrt{2})$$

$$= -1.6568\dots$$

7 b ii $c = -(29 + 16 \cos 45^\circ)$
 $= -(29 + 8\sqrt{2})$
 $= -40.313\dots$
 $x = 7.23$ (3 s.f.)
 (The other value of x is less than -2 .)
 Area $= \frac{1}{2}ac \sin B$
 $= \frac{1}{2} \times 4 \times 9.23 \times \sin 45^\circ$
 $= 13.05\dots$
 $= 13.1 \text{ cm}^2$ (3 s.f.)

8 a



Using $b^2 = a^2 + c^2 - 2ac \cos B$ where $\cos B = \frac{5}{8}$

$$(x+1)^2 = 6^2 + 2^2 - 2 \times 6 \times 2 \times \frac{5}{8}$$

$$x^2 + 2x + 1 = 36 + 4 - 15$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

So $x = 4$ ($x > -1$)

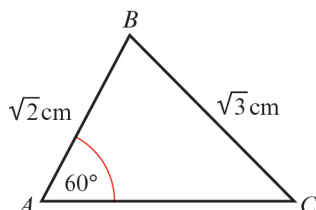
b Use identity, $\cos^2 x + \sin^2 x = 1$.

$$\cos B = \frac{5}{8}$$

so $\sin \angle ABC = \frac{\sqrt{39}}{8}$

Area $= \frac{1}{2}ac \sin B$
 $= \frac{1}{2} \times 6 \times 2 \times \frac{\sqrt{39}}{8}$
 $= 4.68 \text{ cm}^2$ (3 s.f.)

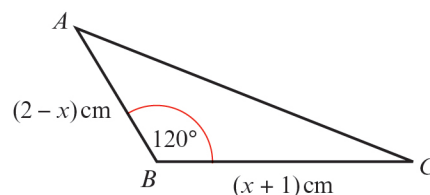
9



9 Using $\frac{\sin C}{c} = \frac{\sin A}{a}$
 $\sin C = \frac{\sqrt{2} \sin 60^\circ}{\sqrt{3}}$
 $= 0.7071\dots$
 $C = \sin^{-1} \left(\frac{\sqrt{2} \sin 60^\circ}{\sqrt{3}} \right)$
 $= 45^\circ$
 $B = 180^\circ - (60 + 45)^\circ$
 $= 75^\circ$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$
 $\frac{AC}{\sin 75^\circ} = \frac{\sqrt{3}}{\sin 60^\circ}$
 So $AC = \frac{\sqrt{3} \sin 75^\circ}{\sin 60^\circ}$
 $= 1.93 \text{ cm}$ (3 s.f.)

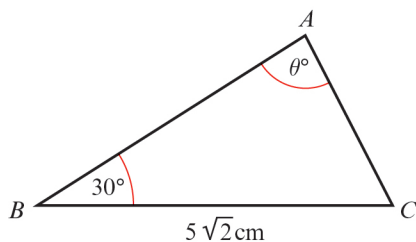
10



a Using the cosine rule:
 $b^2 = a^2 + c^2 - 2ac \cos B$
 $AC^2 = (x+1)^2 + (2-x)^2 - 2(x+1)(2-x) \cos 120^\circ$
 $AC^2 = (x^2 + 2x + 1) + (4 - 4x + x^2) + (x+1)(2-x)$
 $AC^2 = x^2 + 2x + 1 + 4 - 4x + x^2 - x^2 + 2x - x + 2$
 $AC^2 = x^2 - x + 7$

b Completing the square:
 $x^2 - x + 7 = \left(x - \frac{1}{2}\right)^2 + 7 - \frac{1}{4}$
 $= \left(x - \frac{1}{2}\right)^2 + 6\frac{3}{4}$
 This is a minimum when $x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$.

11



Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{AC}{\sin 30^\circ} = \frac{5\sqrt{2}}{\sin \theta^\circ}$$

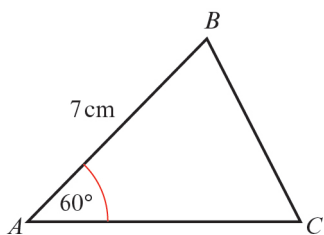
$$AC = \frac{5\sqrt{2} \sin 30^\circ}{\left(\frac{\sqrt{5}}{8}\right)}$$

$$AC = \frac{5\sqrt{2} \sin 30^\circ \times 8}{\sqrt{5}}$$

$$= (\sqrt{5}\sqrt{2})(8 \sin 30^\circ)$$

$$= 4\sqrt{10} \text{ cm}$$

12



Using the cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

with $a = x$, $b = (8 - x)$, $c = 7$ and $A = 60^\circ$

$$x^2 = (8 - x)^2 + 49 - 2(8 - x) \times 7 \times \cos 60^\circ$$

$$x^2 = 64 - 16x + x^2 + 49 - 7(8 - x)$$

$$x^2 = 64 - 16x + x^2 + 49 - 56 + 7x$$

$$\Rightarrow 9x = 57$$

$$\Rightarrow x = \frac{57}{9} = \frac{19}{3} = 6\frac{1}{3}$$

So $BC = 6\frac{1}{3}$ cm and

$$AC = (8 - 6\frac{1}{3}) \text{ cm}$$

$$= 1\frac{2}{3} \text{ cm}$$

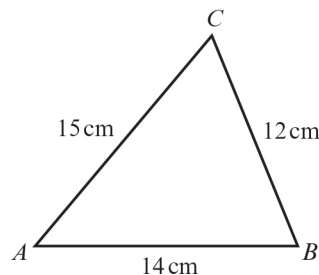
Area $= \frac{1}{2}bc \sin A$

$$= \frac{1}{2} \times 7 \times \frac{5}{3} \times \sin 60^\circ$$

$$= 5.0518\dots$$

$$= 5.05 \text{ cm}^2 \text{ (3 s.f.)}$$

13 a



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{12^2 + 15^2 - 14^2}{2(12)(15)}$$

$$\cos C = \frac{144 + 225 - 196}{360}$$

$$C = 61.278\dots$$

$$C = 61.3^\circ \text{ (3 s.f.)}$$

b Use the formula.

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$= \frac{1}{2} \times 12 \times 15 \times \sin 61.3^\circ$$

$$= 78.943\dots$$

$$= 78.9 \text{ cm}^2 \text{ (3 s.f.)}$$

14 a

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{2.1^2 + 4.2^2 - 5.9^2}{2(2.1)(4.2)}$$

$$\cos A = \frac{4.41 + 17.64 - 34.81}{17.64}$$

$$A = 136.33\dots$$

\therefore Angle $DAB = 136.3^\circ$ (1 d.p.)

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{3.5^2 + 7.5^2 - 5.9^2}{2(3.5)(7.5)}$$

$$\cos C = \frac{12.25 + 56.25 - 34.81}{52.5}$$

$$C = 50.080\dots$$

\therefore Angle $BCD = 50.1^\circ$

b Area $ABD = \frac{1}{2}bc \sin A$

$$= \frac{1}{2} \times 2.1 \times 4.2 \times \sin 136.3^\circ$$

$$= 3.046 \text{ 79}\dots$$

14 b Area $BCD = \frac{1}{2}ab \sin C$
 $= \frac{1}{2} \times 3.5 \times 7.5 \times \sin 50.1^\circ$
 $= 10.069\ 04\dots$

Total area = $3.046\ 79 + 10.069\ 04$
 $= 13.11583$
 \therefore The area of the flower bed is $13.1\ \text{m}^2$.

c First find angle ADB :

$$\cos D = \frac{a^2 + b^2 - d^2}{2ab}$$

$$\cos D = \frac{5.9^2 + 2.1^2 - 4.2^2}{2(5.9)(2.1)}$$

$$\cos D = \frac{34.81 + 4.41 - 17.64}{24.78}$$

So $D = 29.440\ 849\dots$

Now find angle BDC :

$$\cos D = \frac{b^2 + c^2 - d^2}{2bc}$$

$$\cos D = \frac{3.5^2 + 5.9^2 - 7.5^2}{2(3.5)(5.9)}$$

$$\cos D = \frac{12.25 + 34.81 - 56.25}{41.3}$$

So $D = 102.856\ 97\dots$

Angle $ADC = 29.440849 + 102.85697$
 $= 132.298^\circ$

Now find the length AC :

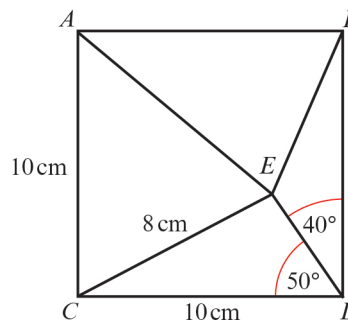
$$d^2 = a^2 + c^2 - 2ac \cos D$$

$$d^2 = 3.5^2 + 2.1^2 - 2 \times 3.5 \times 2.1 \times \cos 132.298^\circ$$

$$d^2 = 12.25 + 4.41 + 9.8929$$

So $d = 5.15$
 So the length of AC is $5.15\ \text{m}$.

15



Use the sine rule to work out angle CED .

$$\frac{\sin E}{e} = \frac{\sin D}{d}$$

$$\frac{\sin E}{10} = \frac{\sin 50^\circ}{8}$$

$$\sin E = \frac{10 \sin 50^\circ}{8}$$

$E = 73.246\ 86^\circ$ or $106.753\ 14^\circ$

The angle is obtuse so

Angle $CED = 106.753\ 14^\circ$

Angle $ECD = 180^\circ - 50^\circ - 106.753\ 14^\circ$
 $= 23.25^\circ$

Use trigonometry to work out the height of triangle CDE .

$$\sin 23.25^\circ = \frac{\text{height}}{8}$$

Height = $3.1575\ \text{cm}$

The height of triangle $ABE = 10 - 3.1575$
 $= 6.84\ \text{cm}$

Area of triangle = $\frac{1}{2} \times 10 \times 6.84 = 34.2$

\therefore Area of the shaded triangle is $34.2\ \text{cm}^2$.