

Review Exercise 1

1 a $8^{\frac{1}{3}}$

$$\begin{aligned} \text{Use } a^{\frac{1}{m}} &= \sqrt[m]{a}, \text{ so } a^{\frac{1}{3}} = \sqrt[3]{a} \\ &= \sqrt[3]{8} \\ &= 2 \end{aligned}$$

b $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} \left(\text{Use } a^{-m} = \frac{1}{a^m} \right)$

$$\text{First find } 8^{\frac{2}{3}} \quad a^{\frac{n}{m}} = \sqrt[m]{(a^n)} \text{ or } (\sqrt[m]{a})^n$$

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$$

$$8^{\frac{2}{3}} = 2^2 = 4$$

$$\begin{aligned} 8^{-\frac{2}{3}} &= \frac{1}{8^{\frac{2}{3}}} \\ &= \frac{1}{4} \end{aligned}$$

2 a $125^{\frac{4}{3}}$

$$a^{\frac{n}{m}} = \sqrt[m]{(a^n)} \text{ or } (\sqrt[m]{a})^n$$

$$= (\sqrt[3]{125})^4$$

$$= 5^4$$

$$= 625$$

b $24x^2 \div 18x^{\frac{4}{3}}$

$$(\text{Use } a^m \div a^n = a^{m-n})$$

$$= \frac{24x^2}{18x^{\frac{4}{3}}} = \frac{4x^2}{3x^{\frac{4}{3}}} \quad \text{cancelling by 6}$$

$$= \frac{4x^{\frac{2}{3}}}{3} \quad \text{because } 2 - \frac{4}{3} = \frac{2}{3}$$

3 a $\sqrt{80}$

$$\begin{aligned} \text{Use } \sqrt{bc} &= \sqrt{b}\sqrt{c} \\ &= \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5} \quad (a=4) \end{aligned}$$

b $(4-\sqrt{5})^2 = (4-\sqrt{5})(4-\sqrt{5})$

$$\begin{aligned} &= 4(4-\sqrt{5}) - \sqrt{5}(4-\sqrt{5}) \\ &= 16 - 4\sqrt{5} - 4\sqrt{5} + 5 \\ &= 21 - 8\sqrt{5} \\ &(b=21 \text{ and } c=-8) \end{aligned}$$

4 a $(4+\sqrt{3})(4-\sqrt{3})$

$$\begin{aligned} &= 4(4-\sqrt{3}) + \sqrt{3}(4-\sqrt{3}) \\ &= 16 - 4\sqrt{3} + 4\sqrt{3} - 3 \\ &= 13 \end{aligned}$$

b $\frac{26}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{26(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}$

$$\begin{aligned} &= \frac{26(4-\sqrt{3})}{13} \\ &= 2(4-\sqrt{3}) \\ &= 8 - 2\sqrt{3} \\ &(a=8 \text{ and } b=-2) \end{aligned}$$

5 a mean = $\frac{1-\sqrt{k}+2+5\sqrt{k}+2\sqrt{k}}{3}$

$$\begin{aligned} &= \frac{3+6\sqrt{k}}{3} \\ &= 1+2\sqrt{k} \end{aligned}$$

$$\begin{aligned} 5 \text{ b } \text{range} &= 2 + 5\sqrt{k} - (1 - \sqrt{k}) \\ &= 1 + 6\sqrt{k} \end{aligned}$$

$$\begin{aligned} 6 \text{ a } y^{-1} &= \left(\frac{1}{25}x^4\right)^{-1} \\ &= \frac{1}{\frac{1}{25}x^4} \\ &= \frac{25}{x^4} \\ &= 25x^{-4} \end{aligned}$$

$$\begin{aligned} \text{b } 5y^{\frac{1}{2}} &= 5\left(\frac{1}{25}x^4\right)^{\frac{1}{2}} \\ &= 5\left(\frac{1}{5}x^2\right) \\ &= x^2 \end{aligned}$$

$$\begin{aligned} 7 \quad \text{Area} &= \frac{1}{2}h(a+b) \\ &= \frac{1}{2}(2\sqrt{2})(3 + \sqrt{2} + 5 + 3\sqrt{2}) \\ &= \sqrt{2}(8 + 4\sqrt{2}) \\ &= 8\sqrt{2} + 8 \end{aligned}$$

The area of the trapezium is $8 + 8\sqrt{2} \text{ cm}^2$.

$$\begin{aligned} 8 \quad \frac{p+q}{p-q} &= \frac{(3-2\sqrt{2}) + (2-\sqrt{2})}{(3-2\sqrt{2}) - (2-\sqrt{2})} \\ &= \frac{5-3\sqrt{2}}{1-\sqrt{2}} \\ &= \frac{(5-3\sqrt{2})}{(1-\sqrt{2})} \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} \\ &= \frac{5+5\sqrt{2}-3\sqrt{2}-6}{1+\sqrt{2}-\sqrt{2}-2} \\ &= \frac{-1+2\sqrt{2}}{-1} \\ &= 1-2\sqrt{2} \quad (m=1, n=-2) \end{aligned}$$

$$9 \text{ a } x^2 - 10x + 16 = (x-8)(x-2)$$

$$\begin{aligned} \text{b } \text{Let } x &= 8^y \\ 8^{2y} - 10(8^y) + 16 &= (8^y - 8)(8^y - 2) = 0 \\ \text{So } 8^y &= 8 \text{ or } 8^y = 2 \\ y &= 1 \text{ or } y = \frac{1}{3} \end{aligned}$$

$$10 \text{ a } x^2 - 8x = (x-4)^2 - 16$$

Complete the square for $x^2 - 8x - 29$

$$\begin{aligned} x^2 - 8x - 29 &= (x-4)^2 - 16 - 29 \\ &= (x-4)^2 - 45 \\ (a &= -4 \text{ and } b = -45) \end{aligned}$$

$$\begin{aligned} \text{b } x^2 - 8x - 29 &= 0 \\ (x-4)^2 - 45 &= 0 \end{aligned}$$

Use the result from part a:

$$(x-4)^2 = 45$$

Take the square root of both sides:

$$x-4 = \pm\sqrt{45}$$

$$x = 4 \pm \sqrt{45}$$

$$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$

$$\text{since } \sqrt{(ab)} = \sqrt{a}\sqrt{b}$$

$$\text{Roots are } 4 \pm 3\sqrt{5}$$

$$(c = 4 \text{ and } d = \pm 3)$$

$$11 \quad f(a) = a(a-2) \text{ and } g(a) = a+5$$

$$a(a-2) = a+5$$

$$a^2 - 2a - a - 5 = 0$$

$$a^2 - 3a - 5 = 0$$

Using the quadratic formula:

$$a = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{29}}{2}$$

$$= 4.19 \text{ or } -1.19$$

As $a > 0$, $a = 4.19$ (3 s.f.)

- 12 a** The height of the athlete's shoulder above the ground is 1.7 m

b $1.7 + 10t - 5t^2 = 0$

Using the quadratic formula when $a = -5$, $b = 10$ and $c = 1.7$

$$t = \frac{-10 \pm \sqrt{(10)^2 - 4(-5)(1.7)}}{2(-5)}$$

$$= \frac{-10 \pm \sqrt{134}}{-10}$$

$$= -0.16 \text{ or } 2.16$$

As $t > 0$, $t = 2.16 \text{ s}$ (3 s.f.)

c $1.7 + 10t - 5t^2 = 1.7 - 5(t^2 - 2t)$

$$= 1.7 - 5((t-1)^2 - 1)$$

$$= 1.7 - 5(t-1)^2 + 5$$

$$= 6.7 - 5(t-1)^2$$

$A = 6.7$, $B = 5$ and $C = 1$

- d** Maximum when $(t-1) = 0$, $t = 1 \text{ s}$ and maximum height = 6.7 m

13 a $f(x) = x^2 - 6x + 18$

$$x^2 - 6x = (x-3)^2 - 9$$

Complete the square for $x^2 - 6x + 18$

$$x^2 - 6x + 18 = (x-3)^2 - 9 + 18$$

$$= (x-3)^2 + 9$$

$a = 3$ and $b = 9$

b $y = x^2 - 6x + 18$

$$y = (x-3)^2 + 9$$

$$(x-3)^2 \geq 0$$

Squaring a number cannot give a negative result.

The minimum value of $(x-3)^2$ is 0, when $x = 3$.

So the minimum value of y is $0 + 9 = 9$, when $x = 3$.

Q is the point (3, 9).

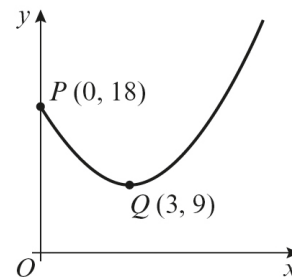
The curve crosses the y -axis where $x = 0$.

- 13 b** For $x = 0$, $y = 18$

P is the point (0, 18).

The graph of $y = x^2 - 6x + 18$ is a \cup shape.

For $y = ax^2 + bx + c$, if $a > 0$, the shape is \cup .



Use the information about P and Q to sketch the curve $x \geq 0$, so the part where $x < 0$ is not needed.

c $y = (x-3)^2 + 9$

Put $y = 41$ into the equation of C .

$$41 = (x-3)^2 + 9$$

Subtract 9 from both sides.

$$32 = (x-3)^2$$

$$(x-3)^2 = 32$$

Take the square root of both sides.

$$x-3 = \pm\sqrt{32}$$

$$x = 3 \pm \sqrt{32}$$

$$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$$

$$\text{using } \sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$x = 3 \pm 4\sqrt{2}$$

x -coordinate of R is $3 + 4\sqrt{2}$

The other value is $3 - 4\sqrt{2}$ which is less than 0, so is not needed.

- 14 a** Using the discriminant

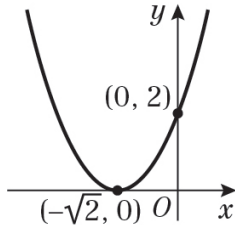
$$b^2 - 4ac = 0 \text{ for equal roots}$$

$$(2\sqrt{2})^2 - 4(1)(k) = 0$$

$$8 - 4k = 0$$

$$k = 2$$

14 b $y = x^2 + 2\sqrt{2}x + 2$
 $= (x + \sqrt{2})^2$
 When $y = 0$, $(x + \sqrt{2})^2 = 0$
 $x = -\sqrt{2}$
 When $x = 0$, $y = 2$



15 a $g(x) = x^9 - 7x^6 - 8x^3$
 $= x^3(x^6 - 7x^3 - 8)$
 To factorise $x^6 - 7x^3 - 8$, let $y = x^3$
 $y^2 - 7y - 8 = (y + 1)(y - 8)$
 So $g(x) = x^3(x^3 + 1)(x^3 - 8)$
 $a = 1$, $b = -8$

b $g(x) = x^3(x^3 + 1)(x^3 - 8) = 0$
 $x^3 = 0$, $x^3 = -1$ or $x^3 = 8$
 $x = -1$, $x = 0$ or $x = 2$

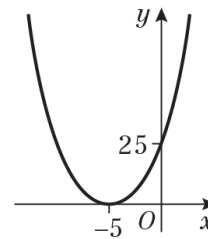
16 a $x^2 + 10x + 36$
 $x^2 + 10x = (x + 5)^2 - 25$
 Complete the square for $x^2 + 10x + 36$
 $x^2 + 10x + 36 = (x + 5)^2 - 25 + 36$
 $= (x + 5)^2 + 11$
 $a = 5$ and $b = 11$

b $x^2 + 10x + 36 = 0$
 $(x + 5)^2 + 11 = 0$
 'Hence' implied in part **a** must be used
 $(x + 5)^2 = -11$
 A real number squared cannot be negative. There are no real roots.

c $x^2 + 10x + k = 0$
 $a = 1$, $b = 10$, $c = k$

16 c For equal roots, $b^2 = 4ac$
 $10^2 = 4 \times 1 \times k$
 $4k = 100$
 $k = 25$

d The graph of $x^2 + 10x + 25$ is a \cup shape.
 For $y = ax^2 + bx + c$, if $a > 0$, the shape is \cup .
 $x = 0$: $y = 0 + 0 + 25 = 25$
 Meets y -axis at $(0, 25)$.
 $y = 0$: $x^2 + 10x + 25 = 0$
 $(x + 5)(x + 5) = 0$
 $x = -5$
 Meets x -axis at $(-5, 0)$.



The graph meets the x -axis at just one point, so it 'touches' the x -axis.

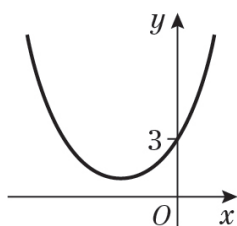
17 a $x^2 + 2x + 3$
 $x^2 + 2x = (x + 1)^2 - 1$
 Complete the square for $x^2 + 2x + 3$
 $x^2 + 2x + 3 = (x + 1)^2 - 1 + 3$
 $= (x + 1)^2 + 2$
 $a = 1$ and $b = 2$

b The graph of $y = x^2 + 2x + 3$ is a \cup shape.
 For $y = ax^2 + bx + c$, if $a > 0$, the shape is \cup .
 $x = 0$: $y = 0 + 0 + 3$
 Put $x = 0$ to find the intersection with the y -axis:
 Meets y -axis at $(0, 3)$.

- 17 b** Put $y = 0$ to find the intersection with the x -axis:

$$\begin{aligned} y = 0 : x^2 + 2x + 3 &= 0 \\ (x + 1)^2 + 2 &= 0 \\ (x + 1)^2 &= -2 \end{aligned}$$

A real number squared cannot be negative, therefore, no real roots, so not intersection with the x -axis.



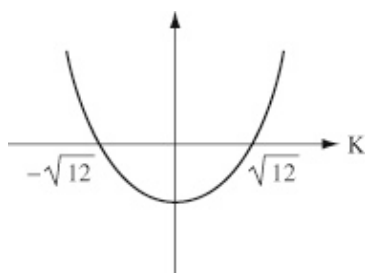
c $x^2 + 2x + 3$
 $a = 1, b = 2, c = 3$
 $b^2 - 4ac = 2^2 - 4 \times 1 \times 3$
 $= -8$

Since the discriminant is negative, the equation has no real roots, so the graph does not cross the x -axis.

d $x^2 + kx + 3 = 0$
 $a = 1, b = k, c = 3$
 For no real roots, $b^2 < 4ac$
 $k^2 < 12$
 $k^2 - 12 < 0$

$$(k + \sqrt{12})(k - \sqrt{12}) < 0$$

This is a quadratic inequality with critical values $-\sqrt{12}$ and $\sqrt{12}$.



Critical values:

$$\begin{aligned} k &= -\sqrt{12}, k = \sqrt{12} \\ -\sqrt{12} &< k < \sqrt{12} \end{aligned}$$

- 17 d** The surds can be simplified

$$\begin{aligned} \text{using } \sqrt{(ab)} &= \sqrt{a}\sqrt{b} \\ \sqrt{12} &= \sqrt{4} \times \sqrt{3} = 2\sqrt{3} \\ (-2\sqrt{3} < k < 2\sqrt{3}) \end{aligned}$$

18 a $2x^2 - x(x - 4) = 8$

$$2x^2 - x^2 + 4x = 8$$

$$x^2 + 4x - 8 = 0$$

b $x^2 + 4x - 8 = 0$

$$x^2 + 4x = (x + 2)^2 - 4$$

$$(x + 2)^2 - 4 - 8 = 0$$

$$(x + 2)^2 = 12$$

$$x + 2 = \pm\sqrt{12}$$

$$x = -2 \pm \sqrt{12}$$

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

$$x = -2 \pm 2\sqrt{3}$$

$$a = -2 \text{ and } b = 2$$

$$\text{Using } y = x - 4:$$

$$y = (-2 \pm 2\sqrt{3}) - 4$$

$$= -6 \pm 2\sqrt{3}$$

$$\text{Solution: } x = -2 \pm 2\sqrt{3}$$

$$y = -6 \pm 2\sqrt{3}$$

19 a $3(2x + 1) > 5 - 2x$

$$6x + 3 > 5 - 2x$$

$$6x + 2x + 3 > 5$$

$$8x > 2$$

$$x > \frac{1}{4}$$

b $2x^2 - 7x + 3 = 0$

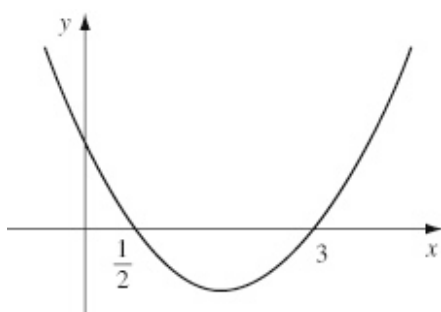
$$(2x - 1)(x - 3) = 0$$

$$(2x - 1) = 0$$

$$(x - 3) = 0$$

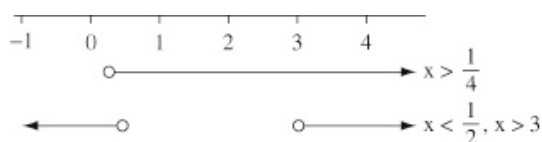
$$x = \frac{1}{2} \text{ or } x = 3$$

19 b



$$2x^2 - 7x + 3 > 0 \text{ where} \\ x < \frac{1}{2} \text{ or } x > 3$$

c



$$\frac{1}{4} < x < \frac{1}{2} \text{ or } x > 3$$

20

$$\begin{aligned} -2(x+1) &= x^2 - 5x + 2 \\ -2x - 2 &= x^2 - 5x + 2 \\ x^2 - 3x + 4 &= 0 \end{aligned}$$

Using the discriminant

$$b^2 - 4ac = (-3)^2 - 4(1)(4) = -7$$

As $b^2 - 4ac < 0$, there are no real roots.
Hence there is no value of x for which $p(x) = q(x)$.

21 a $y = 5 - 2x$

$$2x^2 - 3x - (5 - 2x) = 16$$

$$2x^2 - 3x - 5 + 2x = 16$$

$$2x^2 - x - 21 = 0$$

$$(2x - 7)(x + 3) = 0$$

$$x = 3\frac{1}{2}, x = -3$$

$$x = 3\frac{1}{2}: y = 5 - 7 = -2$$

$$x = -3: y = 5 + 6 = 11$$

$$\text{Solution } x = 3\frac{1}{2}, y = -2$$

$$\text{and } x = -3, y = 11$$

21 b The equation in part a could be written as $y = 5 - 2x$ and $y = 2x^2 - 3x - 16$.

Therefore, the solution to $2x^2 - 3x - 16 = 5 - 2x$ are the same as for part a.

These are the critical values for

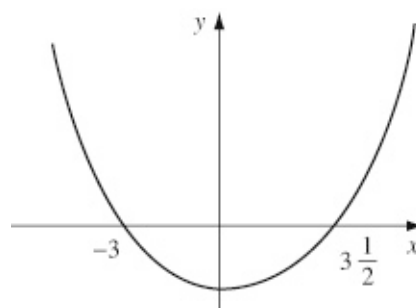
$$2x^2 - 3x - 16 > 5 - 2x:$$

$$x = 3\frac{1}{2} \text{ and } x = -3.$$

$$2x^2 - 3x - 16 > 5 - 2x$$

$$(2x^2 - 3x - 16 - 5 + 2x > 0)$$

$$2x^2 - x - 21 > 0$$



$$x < -3 \text{ or } x > 3\frac{1}{2}$$

22 a $x^2 + kx + (k+3) = 0$

$$a = 1, b = k, c = k + 3$$

$$b^2 > 4ac$$

$$k^2 > 4(k+3)$$

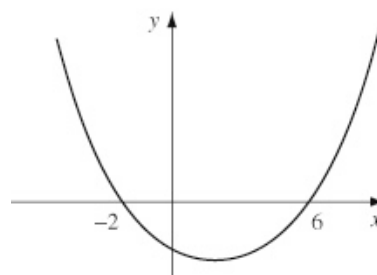
$$k^2 > 4k + 12$$

$$k^2 - 4k - 12 > 0$$

b $k^2 - 4k - 12 = 0$

$$(k+2)(k-6) = 0$$

$$k = -2, k = 6$$



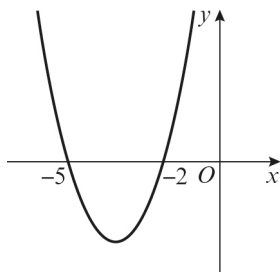
$$k^2 - 4k - 12 > 0 \text{ where } k < -2 \text{ or } k > 6$$

23 $\frac{6}{x+5} < 2$

Multiply both sides by $(x+5)^2$
 $6(x+5) < 2(x+5)^2$
 $6x+30 < 2x^2+20x+50$
 $2x^2+14x+20 > 0$

Solve the quadratic to find the critical values.

$2x^2+14x+20=0$
 $2(x^2+7x+10)=0$
 $2(x+5)(x+2)=0$
 $x=-5$ or $x=-2$



The solution is $x < -5$ or $x > -2$.

24 a $9 - x^2 = 0$
 $(3+x)(3-x) = 0$
 $x = -3$ or $x = 3$
 When $x = 0$, $y = 9$

To work out the points of intersection, solve the equations simultaneously.

$9 - x^2 = 14 - 6x$
 $x^2 - 6x + 5 = 0$
 $(x-5)(x-1) = 0$
 $x = 1$ or $x = 5$

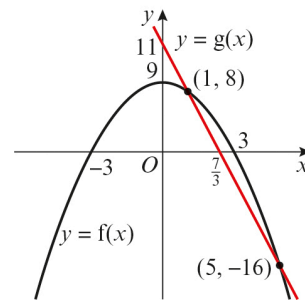
When $x = 1$, $y = 8$
 When $x = 5$, $y = -16$

Let $14 - 6x = 0$

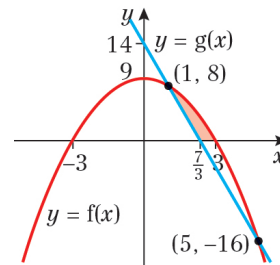
$x = \frac{14}{6} = \frac{7}{3}$

The line crosses the x -axis at $\left(\frac{7}{3}, 0\right)$.

24 a



b



25 a $x^3 - 4x = x(x^2 - 4)$
 $= x(x+2)(x-2)$

b Curve crosses the x -axis where $y = 0$

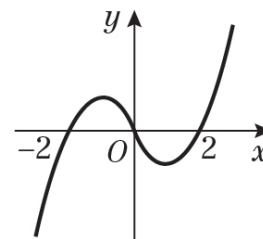
$x(x+2)(x-2) = 0$

$x = 0$, $x = -2$, $x = 2$

When $x = 0$, $y = 0$

When $x \rightarrow \infty$, $y \rightarrow \infty$

When $x \rightarrow -\infty$, $y \rightarrow -\infty$



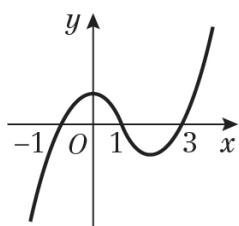
Crosses the y -axis at $(0, 0)$.

Crosses the x -axis at $(-2, 0)$, $(2, 0)$.

c $y = x^3 - 4x$
 $y = (x-1)^3 - 4(x-1)$

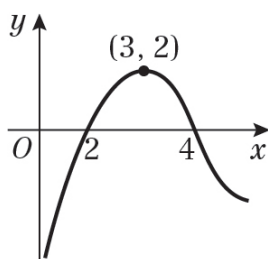
This is a translation of +1 in the x -direction.

25 c



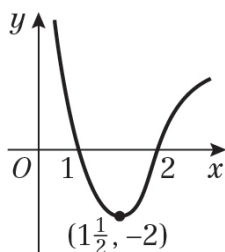
Crosses the x -axis at $(-1, 0)$, $(1, 0)$ and $(3, 0)$.

26 a



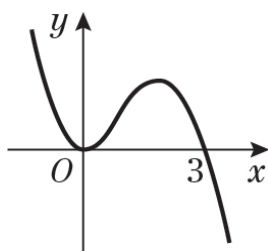
Crosses the x -axis at $(2, 0)$, $(4, 0)$.
Image of P is $(3, 2)$.

26 b



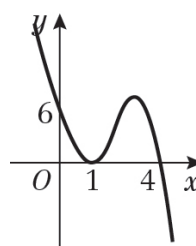
Crosses the x -axis at $(1, 0)$, $(2, 0)$.
Image of P is $(1\frac{1}{2}, -2)$.

27 a



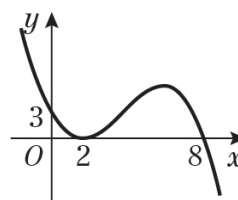
Meets the y -axis at $(0, 0)$.
Meets the x -axis at $(0, 0)$, $(3, 0)$.

27 b



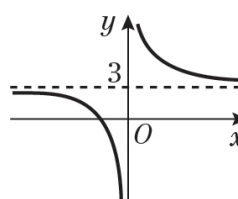
Meets the y -axis at $(0, 6)$.
Meets the x -axis at $(1, 0)$, $(4, 0)$.

c



Meets the y -axis at $(0, 3)$.
Meets the x -axis at $(2, 0)$, $(8, 0)$.

28 a



$y = 3$ is an asymptote.
 $x = 0$ is an asymptote.

b The graph does not cross the y -axis
(see sketch in part a).

Crosses the x -axis where $y = 0$:

$$\frac{1}{x} + 3 = 0$$

$$\frac{1}{x} = -3$$

$$x = -\frac{1}{3}, \left(-\frac{1}{3}, 0\right)$$

29 a $(x^2 - 5x + 2)(x^2 - 5x + 4) = 0$
For $x^2 - 5x + 2 = 0$

Using the quadratic formula:

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{17}}{2}$$

$x = 4.56$ or $x = 0.438$

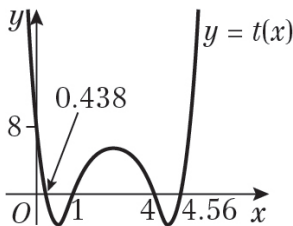
For $x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x = 1$ or $x = 4$

$x = 0.438, 1, 4$ or 4.56

b When $x = 0, y = 8$



30 a $y = -f(x)$ is a reflection in the x -axis of $y = f(x)$, so P is transformed to $(6, 8)$.

b $y = f(x - 3)$ is a translation 3 units to the right of $y = f(x)$, so P is transformed to $(9, -8)$.

c $2y = f(x)$ is $y = \frac{1}{2}f(x)$ which is a vertical stretch scale factor $\frac{1}{2}$ of $y = f(x)$, so P is transformed to $(6, -4)$.

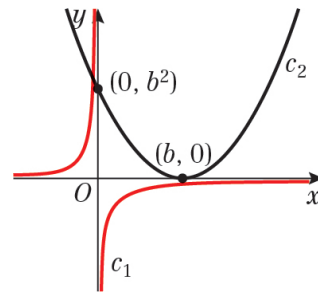
31 a $y = -\frac{a}{x}$ is the curve $y = \frac{k}{x}, k < 0$

$y = (x - b)^2$ is a translation, b units to the right of the curve $y = x^2$

When $x = 0, y = b^2$

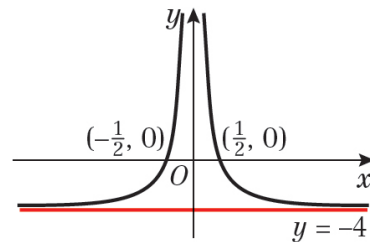
When $y = 0, x = b$

31 a



b The graphs intersect at 1 point, so have 1 point of intersection.

32 a $y = \frac{1}{x^2} - 4$ is a translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ of $y = \frac{1}{x^2}$



b When $y = \frac{1}{(x+k)^2} - 4$ passes through the origin, $x = 0$ and $y = 0$.

So $\frac{1}{k^2} - 4 = 0$

$\frac{1}{k^2} = 4$

$k = \pm \frac{1}{2}$

Challenge

1 a $x^2 - 10x + 9 = 0$
 $(x - 1)(x - 9) = 0$
 $x = 1$ or $x = 9$

b $3^{x-2}(3^x - 10) = -1$
 $3^{2x-2} - 10 \times 3^{x-2} + 1 = 0$
 Multiply by 3^2
 $3^{2x} - 10 \times 3^x + 9 = 0$
 Let $y = 3^x$
 $y^2 - 10y + 9 = 0$
 Using your answers from part a
 $y = 1$ or 9
 $3^x = 1$ or $3^x = 9$
 $x = 0$ or 2

2 Let x and y be the length and width of the rectangle.
 Area = $xy = 6$
 Perimeter = $2x + 2y = 8\sqrt{2}$
 $2y = 8\sqrt{2} - 2x$
 $y = 4\sqrt{2} - x$
 Solving simultaneously
 $x(4\sqrt{2} - x) = 6$
 $x^2 - 4\sqrt{2}x + 6 = 0$

Using the quadratic formula

$$x = \frac{4\sqrt{2} \pm \sqrt{(4\sqrt{2})^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{4\sqrt{2} \pm \sqrt{8}}{2}$$

$$= \frac{4\sqrt{2} \pm 2\sqrt{2}}{2}$$

$$x = \sqrt{2} \text{ or } x = 3\sqrt{2}$$

$$\text{When } x = \sqrt{2}, y = 3\sqrt{2}$$

$$\text{When } x = 3\sqrt{2}, y = \sqrt{2}$$

The dimensions of the rectangle are
 $\sqrt{2}$ cm and $3\sqrt{2}$ cm.

3 Solving simultaneously
 $3x^3 + x^2 - x = 2x(x - 1)(x + 1)$
 $3x^3 + x^2 - x = 2x(x^2 - 1)$
 $3x^3 + x^2 - x = 2x^3 - 2x$
 $x^3 + x^2 + x = 0$
 $x(x^2 + x + 1) = 0$

The discriminant of $x^2 + x + 1$
 $b^2 - 4ac = 1^2 - 4(1)(1) = -3$
 $-3 < 0$ so there are no real solutions for $x^2 + x + 1$

The only solution is when $x = 0$ at $(0, 0)$.

4 $f(x) = (x^2 + x - 20)(x^2 + x - 2)$
 $= (x + 5)(x - 4)(x + 2)(x - 1)$
 when $f(x) = 0$
 $x = -5, -2, 1$ or 4

$$g(x - k) = (x - k + 5)(x - k - 4)(x - k + 2)(x - k - 1)$$

When $k = 3$,

$$g(x - 3) = (x + 2)(x - 7)(x - 1)(x - 4)$$

$(x + 2)$, $(x - 1)$ and $(x - 4)$ match

$$\text{When } k = -3, g(x + 3) = (x + 8)(x - 1)(x + 5)(x + 2)$$

$(x - 1)$, $(x + 5)$ and $(x + 2)$ match
 So $k = -3$ or 3 .

5 $16m^2 = 54\sqrt{m}$
 $\frac{m^2}{\sqrt{m}} = \frac{54}{16}$
 $m^{\frac{3}{2}} = \frac{27}{8}$
 $\left(m^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}}$
 $m = \frac{9}{4}$

- 5 When $m = 0$, $16m^2 = 0$ and $54\sqrt{m}$
So, real solutions to $16m^2 = 54\sqrt{m}$
are $m = \frac{9}{4}$ and $m = 0$