Review exercise 2

1 The equation of the line is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-8}{6-8}=\frac{x+2}{4+2}$$

$$\frac{y-8}{-2}=\frac{x+2}{6}$$

$3y-24=-x-2$

$x+3y-22=0$

2

$$y-(-4)=\tfrac{1}{3}(x-9)$$

$$y+4=\tfrac{1}{3}(x-9)$$

$$3y+12=x-9$$

$$x-3y-21=0$$

$a=1$, $b=-3$, $c=-21$

3 Using points A and B:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-3}{5-3}=\frac{x-0}{k-0}$$

$$\frac{y-3}{2}=\frac{x}{k}$$

$ky-3k=2x$

Substituting point C into the equation:

$k(2k)-3k=2(10)$

$2k^2-3k-20=0$

$(2k+5)(k-4)=0$

$k=-\frac{5}{2}$ or $k=4$

4 a Using points (160, 72) and (180, 81):

$$\text{Gradient}=\frac{y_2-y_1}{x_2-x_1}=\frac{81-72}{180-160}=\frac{9}{20}=0.45$$

b $l=kh$, where k is the gradient.
So $l=0.45h$

c The model may not be valid for young people/children who are still growing.

5 a The gradient of l_1 is 3.
So the gradient of l_2 is $-\frac{1}{3}$.
The equation of line l_2 is:

$$y-2=-\tfrac{1}{3}(x-6)$$

$$y-2=-\tfrac{1}{3}x+2$$

$$y=-\tfrac{1}{3}x+4$$

b

$$y=3x-6$$

$$y=-\tfrac{1}{3}x+4$$

$$3x-6=-\tfrac{1}{3}x+4$$

$$3x+\tfrac{1}{3}x=4+6$$

$$\tfrac{10}{3}x=10$$

$$x=3$$

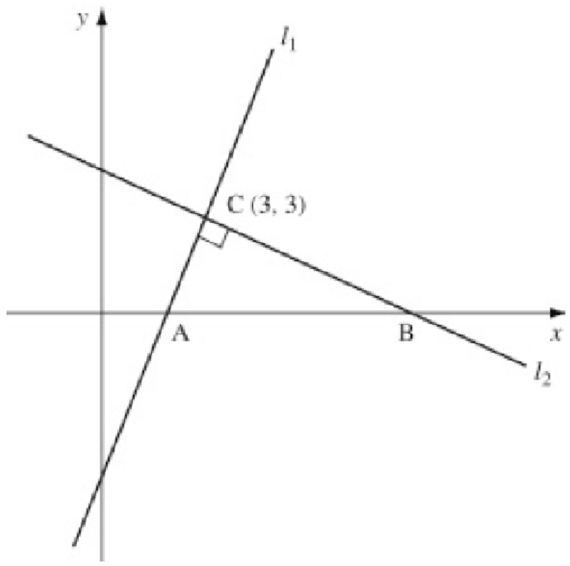
$$y=(3\times 3)-6=3$$

$x=3$

$y=3\times 3-6=3$

The point C is (3, 3).

c



Where l_1 meets the x-axis, $y=0$:

$0=3x-6$

$3x=6$

$x=2$

The point A is (2, 0).

Where l_2 meets the x-axis, $y=0$:

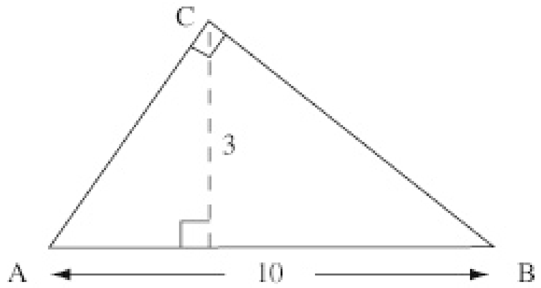
$0=-\frac{1}{3}x+4$

$\frac{1}{3}x=4$

$x=12$

The point B is (12, 0).

5 c



$AB = 12 - 2 = 10$
The perpendicular height, using AB as the base is 3.
Area of triangle $ABC = \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 10 \times 3$
$= 15$

6 Substituting $y = 2x$ into $5y + x - 33 = 0$:
$5(2x) + x - 33 = 0$
$11x - 33 = 0$
$x = 3$
$y = 2 \times 3 = 6$
The point P is (3, 6).
Distance from origin $= \sqrt{3^2 + 6^2}$
$= \sqrt{45}$
$= 3\sqrt{5}$

7 Gradient of line $= \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-4 - 8}{7 - 5}$
$= -6$
Gradient of the perpendicular bisector is $\frac{1}{6}$
Midpoint of line $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{5 + 7}{2}, \frac{8 - 4}{2}\right)$
$= (6, 2)$

Equation of the perpendicular bisector is:
$y - y_1 = m(x - x_1)$
$y - 2 = \frac{1}{6}(x - 6)$
$y = \frac{1}{6}x + 1$

This line crosses the x-axis at $y = 0$:
$\frac{1}{6}x + 1 = 0$
$x = -6$

The point Q is (−6, 0).

8 Equation of circle with centre (−3, 8) and radius r:
$(x + 3)^2 + (y - 8)^2 = r^2$

r = distance from (−3, 8) to (0, 9)
$r^2 = (0 + 3)^2 + (9 - 8)^2 = 9 + 1 = 10$

The equation for C is:
$(x + 3)^2 + (y - 8)^2 = 10$

9 a Rearranging:
$x^2 - 6x + y^2 + 2y = 10$

Completing the square:
$(x - 3)^2 - 9 + (y + 1)^2 - 1 = 10$
$(x - 3)^2 + (y + 1)^2 = 20$
$a = 3, b = -1, r = \sqrt{20}$

b The circle has centre (3, −1) and radius $\sqrt{20}$.

10 a Rearranging $3x + y = 14$:
$y = 14 - 3x$

Solving simultaneously using substitution:
$(x - 2)^2 + (14 - 3x - 3)^2 = 5$
$(x - 2)^2 + (-3x + 11)^2 = 5$
$x^2 - 4x + 4 + 9x^2 - 66x + 121 - 5 = 0$
$10x^2 - 70x + 120 = 0$
$x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
So $x = 3$ and $x = 4$

$x = 3$: $y = 14 - 3 \times 3 = 5$
$x = 4$: $y = 14 - 3 \times 4 = 2$
Point A is (3, 5) and point B is (4, 2).

b Using Pythagoras' theorem:
Length $AB = \sqrt{(4-3)^2 + (2-5)^2}$
$= \sqrt{10}$

11 The equation of the circle is $x^2 + y^2 = r^2$.

Solving simultaneously using substitution:
$x^2 + (3x - 2)^2 = r^2$
$x^2 + 9x^2 - 12x + 4 - r^2 = 0$
$10x^2 - 12x + 4 - r^2 = 0$

Using the discriminant for no solutions:
$b^2 - 4ac < 0$
$(-12)^2 - 4(10)(4 - r^2) < 0$
$144 - 160 + 40r^2 < 0$

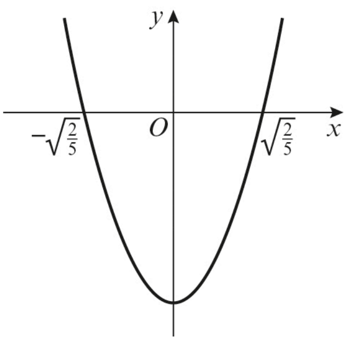
11 $40r^2 - 16 < 0$

When $40r^2 - 16 = 0$

$8(5r^2 - 2) = 0$

$r^2 = \frac{2}{5}$

$r = \pm\sqrt{\frac{2}{5}}$



$-\sqrt{\frac{2}{5}} < r < \sqrt{\frac{2}{5}}$

However, the radius cannot be negative.

So $0 < r < \sqrt{\frac{2}{5}}$

12 a Equation of circle with centre (1, 5) and radius r:

$(x - 1)^2 + (y - 5)^2 = r^2$

r = distance from (1, 5) to (4, −2)

$r^2 = (4 - 1)^2 + (-2 - 5)^2$
$= 9 + 49$
$= 58$

The equation for C is:

$(x - 1)^2 + (y - 5)^2 = 58$

b Gradient of the radius of the circle at P

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-5}{4-1} = -\frac{7}{3}$

Gradient of the tangent $= \frac{3}{7}$

Equation of the tangent at P:

$y - y_1 = m(x - x_1)$

$y + 2 = \frac{3}{7}(x - 4)$

$7y - 3x + 26 = 0$

13 a $AB^2 = (6 - 2)^2 + (5 - 1)^2$
$= 4^2 + 4^2 = 32$

$BC^2 = (8 - 6)^2 + (3 - 5)^2$
$= 2^2 + 2^2 = 8$

13 a $AC^2 = (8 - 2)^2 + (3 - 1)^2$
$= 6^2 + 2^2 = 40$

Using Pythagoras' theorem:

$AB^2 + BC^2 = 32 + 8 = 40 = AC^2$

Therefore, $\angle ABC$ is 90°.

b As triangle ABC is a right-angled triangle, AC is a diameter of the circle.

c AC is a diameter of the circle, so the midpoint of AC is the centre.

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2+8}{2}, \frac{1+3}{2}\right)$

$= (5, 2)$

Radius $= \frac{1}{2} \times AC$

$= \frac{1}{2} \times \sqrt{40}$

$= \frac{1}{2} \times 2\sqrt{10}$

$= \sqrt{10}$

The equation of the circle is:

$(x - 5)^2 + (y - 2)^2 = 10$

14 $\frac{2x^2 + 20x + 42}{224x + 4x^2 - 4x^3} = \frac{x^2 + 10x + 21}{112x + 2x^2 - 2x^3}$

$= \frac{(x+3)(x+7)}{-2x(x^2 - x - 56)}$

$= \frac{(x+3)(x+7)}{-2x(x+7)(x-8)}$

$= \frac{(x+3)}{-2x(x-8)}$

$a = 3, b = -2, c = -8$

15 a Using the factor theorem:

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 - 7(\frac{1}{2})^2 - 17(\frac{1}{2}) + 10$

$= \frac{1}{4} - \frac{7}{4} - \frac{17}{2} + 10$

$= 0$

So $(2x - 1)$ is a factor of $2x^3 - 7x^2 - 17x + 10$.

15 b

$$\begin{array}{r} x^2 - 3x - 10 \\ 2x - 1 \overline{\smash{)}\, 2x^3 - 7x^2 - 17x + 10} \\ \underline{2x^3 - x^2} \\ -6x^2 - 17x \\ \underline{-6x^2 + 3x} \\ -20x + 10 \\ \underline{-20x + 10} \\ 0 \end{array}$$

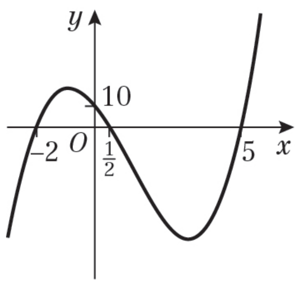
$2x^3 - 7x^2 - 17x + 10$
$= (2x - 1)(x^2 - 3x - 10)$
$= (2x - 1)(x - 5)(x + 2)$

c $(2x - 1)(x - 5)(x + 2) = 0$
So $x = \frac{1}{2}$, $x = 5$ or $x = -2$

So the curve crosses the x-axis at $(\frac{1}{2}, 0)$, $(5, 0)$ and $(-2, 0)$.

When $x = 0$, $y = -1 \times -5 \times 2 = 10$
So the curve crosses the y-axis at $(0, 10)$.

$x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$



16 $f(x) = 3x^3 + x^2 - 38x + c$

a $f(3) = 0$
$3(3)^3 + (3)^2 - 38(3) + c = 0$
$3 \times 27 + 9 - 114 + c = 0$
$c = 24$

b $f(x) = 3x^3 + x^2 - 38x + 24$
$f(3) = 0$, so $(x - 3)$ is a factor of $3x^3 + x^2 - 38x + 24$.

16 c

$$\begin{array}{r} 3x^2 + 10x - 8 \\ x - 3 \overline{\smash{)}\, 3x^3 + x^2 - 38x + 24} \\ \underline{3x^3 - 9x^2} \\ 10x^2 - 38x \\ \underline{10x^2 - 30x} \\ -8x - 24 \\ \underline{-8x + 24} \\ 0 \end{array}$$

$3x^3 + x^2 - 38x + 24$
$= (x - 3)(3x^2 + 10x - 8)$
$= (x - 3)(3x - 2)(x + 4)$

17 a $g(x) = x^3 - 13x + 12$
$g(3) = (3)^3 - 13(3) + 12$
$= 27 - 39 + 12$
$= 0$
So $(x - 3)$ is a factor of $g(x)$.

b

$$\begin{array}{r} x^2 + 3x - 4 \\ x - 3 \overline{\smash{)}\, x^3 - 0x^2 - 13x + 12} \\ \underline{x^3 - 3x^2} \\ 3x^2 - 13x \\ \underline{3x^2 - 9x} \\ -4x + 12 \\ \underline{-4x + 12} \\ 0 \end{array}$$

$g(x) = x^3 - 13x + 12$
$= (x - 3)(x^2 + 3x - 4)$
$= (x - 3)(x + 4)(x - 1)$

18 a Example:
When $a = 0$ and $b = 0$, $0^2 + 0^2 = (0 + 0)^2$.

b $(a + b)^2 = a^2 + 2ab + b^2$
When $a > 0$ and $b > 0$, $2ab > 0$
Therefore $a^2 + b^2 < (a + b)^2$
When $a < 0$ and $b < 0$, $2ab > 0$
Therefore $a^2 + b^2 < (a + b)^2$
When $a > 0$ and $b < 0$, $2ab < 0$
Therefore $a^2 + b^2 > (a + b)^2$
When $a < 0$ and $b > 0$, $2ab < 0$
Therefore $a^2 + b^2 > (a + b)^2$
The conditions are $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$.

19 a $p = 5$: $5^2 = 25 = 24 + 1$
$p = 7$: $7^2 = 49 = 2(24) + 1$
$p = 11$: $11^2 = 121 = 5(24) + 1$
$p = 13$: $13^2 = 169 = 7(24) + 1$
$p = 17$: $17^2 = 289 = 12(24) + 1$
$p = 19$: $19^2 = 361 = 15(24) + 1$

b $3(24) + 1 = 73$ and 73 is not a square number.

20 a Rearranging:
$x^2 - 10x + y^2 - 8y = -32$
Completing the square:
$(x - 5)^2 - 25 + (y - 4)^2 - 16 = -32$
$(x - 5)^2 + (y - 4)^2 = 9$
$(x - 5)^2 + (y - 4)^2 = 3^2$
$a = 5, b = 4, r = 3$

b Centre of circle C is (5, 4).
Centre of circle D is (0, 0).
Using Pythagoras' theorem:
Distance $= \sqrt{(5-0)^2 + (4-0)^2} = \sqrt{41}$

c Radius of circle $C = 3$
Radius of circle $D = 3$
Distance between the centres $= \sqrt{41}$
$3 + 3 < \sqrt{41}$
Therefore, the circles C and D do not touch.

21 a $(1 - 2x)^{10}$
$= 1^{10} + \binom{10}{1}1^9(-2x) + \binom{10}{2}1^8(-2x)^2 + \binom{10}{3}1^7(-2x)^3 + \ldots$
$= 1 + 10(-2x) + \frac{10(9)}{2}(-2x) + \frac{10(9)(8)}{6}(-2x)^3 + \ldots$
$= 1 - 20x + 180x^2 - 960x^3 + \ldots$

b $(0.98)^{10}$
$= (1 - 2(0.01))^{10}$
$= 1 - 20(0.01) + 180(0.01)^2 - 960(0.01)^3 + \ldots$
$= 0.817$ (3 d.p.)

22 $(1 + 2x)^5$
$= 1^5 + \binom{5}{1}1^4(2x) + \binom{5}{2}1^3(2x)^2 + \ldots$
$= 1 + 5(2x) + \frac{5(4)}{2}(2x)^2 + \ldots$
$= 1 + 10x + 40x^2 + \ldots$
$(2 - x)(1 + 2x)^5$
$= (2 - x)(1 + 10x + 40x^2 + \ldots)$
$= 2 + 20x + 80x^2 + \ldots - x - 10x^2 + \ldots$
$= 2 + 19x + 70x^2 + \ldots$
$\approx 2 + 19x + 70x^2$
$a = 2, b = 19, c = 70$

23 $(2 - 4x)^q$
x term $= \binom{q}{q-1}2^{q-1}(-4x)^1$
$= q \times 2^{q-1} \times -4x$
$= -4 \times 2^{q-1}qx$
$-4 \times 2^{q-1}q = -32q$
$2^{q-1} = 8$
$q - 1 = 3$
$q = 4$

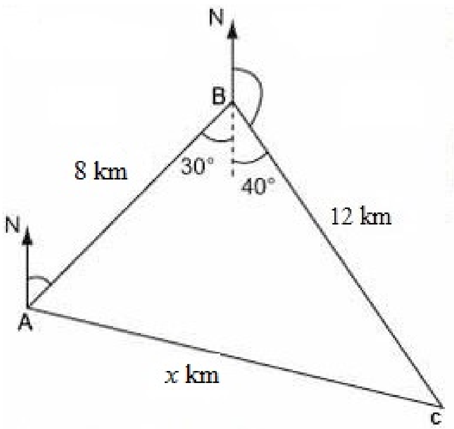
24 Using the sine rule:
$\frac{b}{\sin B} = \frac{c}{\sin C}$
$\frac{b}{\sin 45°} = \frac{\sqrt{5}}{\sin 30°}$
$b = \frac{\sqrt{5}\sin 45°}{\sin 30°}$
$b = \frac{\sqrt{5} \times \frac{\sqrt{2}}{2}}{\frac{1}{2}}$
$b = \sqrt{10}$
$AC = \sqrt{10}$ cm

25 a Using the cosine rule:
$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$\cos 60° = \frac{(2x-3)^2 + 5^2 - (x+1)^2}{2(2x-3)(5)}$
$\frac{1}{2} = \frac{4x^2 - 12x + 9 + 25 - (x^2 + 2x + 1)}{10(2x-3)}$
$5(2x - 3) = 3x^2 - 14x + 33$
$3x^2 - 24x + 48 = 0$
$x^2 - 8x + 16 = 0$

25 b $x^2 - 8x + 16 = 0$
$(x-4)^2 = 0$
$x = 4$

c Area = $ac \sin B$
$a = 2 \times 4 - 3 = 5$
$c = 5$
Area $= \frac{1}{2} \times 5 \times 5 \times \sin 60°$
$= 10.8253...$
$= 10.8\text{ cm}^2$ (3 s.f.)

26



a Using the cosine rule
$x^2 = 8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 70°$
$= 142.332\ldots$
$x = 11.93\text{ km}$
The distance of ship C from ship A is 11.93 km.

b Using the sine rule:
$\frac{\sin 70°}{11.93} = \frac{\sin A}{12}$
$\sin A = 0.94520\ldots$
$A = 70.9°$
The bearing of ship C from ship A is 100.9°.

27 a If triangle ABC is isosceles, then two of the sides are equal.
$AB = \sqrt{(6+2)^2 + (10-4)^2} = \sqrt{100} = 10$
$BC = \sqrt{(16-6)^2 + (10-10)^2} = \sqrt{100} = 10$
$AC = \sqrt{(16+2)^2 + (10-4)^2} = \sqrt{360}$
$= 6\sqrt{10}$
$AB = BC$
Therefore ABC is an isosceles triangle.

27 b Using the cosine rule:
$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$= \frac{10^2 + 10^2 - \left(\sqrt{360}\right)^2}{2(10)(10)}$
$= \frac{100 + 100 - 360}{200}$
$= -\frac{4}{5}$
$B = 143.13010...$
Angle $ABC = 143.1°$ (1 d.p.)

28 Using the sine rule in triangle ABD:
$\frac{\sin \angle BDA}{4.3} = \frac{\sin 40°}{3.5}$
$\sin \angle BDA = \frac{4.3 \sin 40°}{3.5}$
$= 0.78971\ldots$
$\angle BDA = 52.16°$

Using the angle sum of a triangle:
$\angle ABD = 180° - (52.16° + 40°)$
$= 87.84°$

Using the sine rule in triangle ABD:
$\frac{AD}{\sin 87.84} = \frac{3.5}{\sin 40°}$
$AD = 5.44\text{ cm}$

$AC = AD + DC$
$= 5.44 + 8.6$
$= 14.04\text{ cm}$

Area of triangle ABC
$= \frac{1}{2} \times 4.3 \times 14.04 \times \sin 40°$
$= 19.4\text{ cm}^2$

29 a $(x-5)^2 + (y-2)^2 = 5^2$
$(x-5)^2 + (y-2)^2 = 25$

b Substituting $x = 8$ and $y = k$ into the equation of the circle:
$(8-5)^2 + (k-2)^2 = 25$
$9 + k^2 - 4k + 4 - 25 = 0$
$k^2 - 4k - 12 = 0$
$(k+2)(k-6) = 0$
$k = -2$ or $k = 6$
k is positive, therefore $k = 6$.

29 c $XY = \sqrt{(10-1)^2 + (2+1)^2} = \sqrt{90}$

$YZ = \sqrt{(8-10)^2 + (6-2)^2} = \sqrt{20}$

$XZ = \sqrt{(8-1)^2 + (6+1)^2} = \sqrt{98}$

Using the cosine rule:

$\cos Y = \dfrac{x^2 + z^2 - y^2}{2xz}$

$= \dfrac{20+90-98}{2\sqrt{1800}}$

$= \dfrac{12}{60\sqrt{2}}$

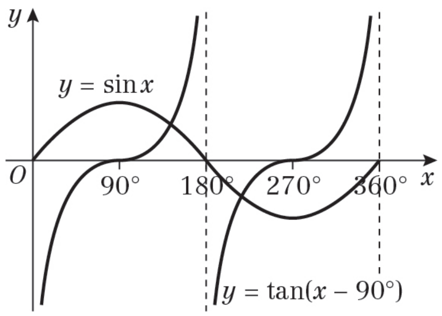
$= \dfrac{1}{5\sqrt{2}}$

Rationalising the denominator:

$\cos Y = \dfrac{\sqrt{2}}{10}$

So $\cos \angle XYZ = \dfrac{\sqrt{2}}{10}$

30 a



b There are two solutions in the interval $0 \leq x \leq 360°$.

31 a The curve $y = \sin x$ crosses the x-axis at (−360°, 0), (−180°, 0), (0°, 0), (180°, 0) and (360°, 0).
$y = \sin(x + 45°)$ is a translation of $\begin{pmatrix} -45° \\ 0 \end{pmatrix}$
so subtract 45° from the x-coordinates.

The curve crosses the x-axis at (−405°, 0), (−225°, 0), (−45°, 0), (135°, 0) and (315°, 0).
(−405°, 0) is not in the range, so (−225°, 0), (−45°, 0), (135°, 0) and (315°, 0)

31 b The curve $y = \sin(x + 45°)$ crosses the y-axis when $x = 0$.

$\sin 45° = \dfrac{\sqrt{2}}{2}$

$\left(0, \dfrac{\sqrt{2}}{2}\right)$

32 Each of the four triangular faces is an equilateral triangle.

Area of one triangle

$= \frac{1}{2}ac \sin B$

$= \frac{1}{2} \times s \times s \times \sin 60°$

$= \dfrac{s^2}{2} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}s^2}{4}$ cm^2

Total area

= area of 4 triangles + area of square

$= 4 \times \dfrac{\sqrt{3}s^2}{4} + s^2$

$= \sqrt{3}s^2 + s^2$

$= \left(\sqrt{3} + 1\right)s^2$

The total surface area of the pyramid is $\left(\sqrt{3} + 1\right)s^2$ cm^2.

33 a $\sin\theta = \cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = 1$

So $\tan\theta = 1$

b When $\tan\theta = 1$
$\theta = 45°$ or $225°$
So $\sin\theta = \cos\theta$ when $\theta = 45°$ or $225°$

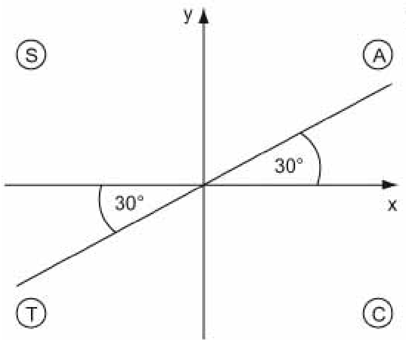
34 $3\tan^2 x = 1$

$\tan x = \pm\dfrac{1}{\sqrt{3}}$

For $\tan x = \dfrac{1}{\sqrt{3}}$

$x = 30°$

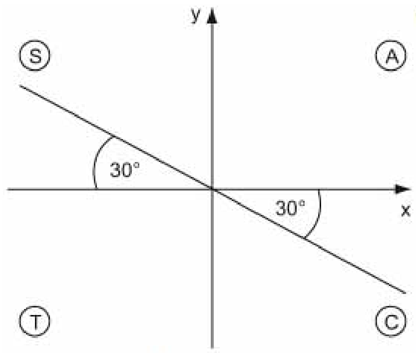
34



So $x = 30°$ or $x = 210°$

For $\tan x = -\frac{1}{\sqrt{3}}$

$x = 330°$ (or $-30°$)

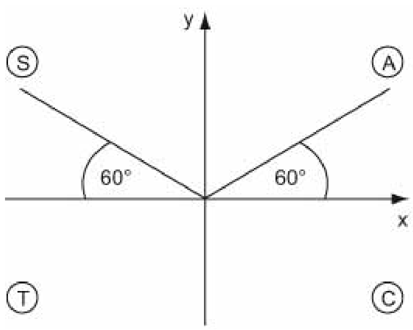


So $x = 330°$ or $x = 150°$
So $x = 30°, 150°, 210°$ or $330°$

35 $2\sin(\theta - 30°) = \sqrt{3}$

$\sin(\theta - 30°) = \frac{\sqrt{3}}{2}$

$\theta - 30° = 60°$



So $\theta - 30° = 60°$ or $\theta - 30° = 120°$
When $\theta - 30° = 60°$
$\theta = 60° + 30°$
$= 90°$
When $\theta - 30° = 120°$
$\theta = 120° + 30°$
$= 150°$
So $\theta = 90°$ or $150°$

36 a

$2\cos^2 x = 4 - 5\sin x$
$2(1 - \sin^2 x) = 4 - 5\sin x$
$2 - 2\sin^2 x = 4 - 5\sin x$
$2\sin^2 x - 5\sin x + 2 = 0$ (as required)

b Let $\sin x = y$
$2y^2 - 5y + 2 = 0$
$(2y - 1)(y - 2) = 0$
So $y = \frac{1}{2}$ or $y = 2$
When $\sin x = \frac{1}{2}$, $x = 30°$
or $x = 180° - 30° = 150°$
$\sin x = 2$ is impossible.
$x = 30°$ or $150°$

37

$2\tan^2 x - 4 = 5\tan x$
$2\tan^2 x - 5\tan x - 4 = 0$

Using the quadratic formula:

$$\tan x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{57}}{4}$$

When $\tan x = \frac{5 + \sqrt{57}}{4}$, $x = 72.3°$
or $x = 72.3° + 180° = 252.3°$
When $\tan x = \frac{5 - \sqrt{57}}{4}$, $x = -32.5°$
or $x = -32.5° + 180° = 147.5°$
or $x = 147.5° + 180° = 327.5°$
$x = 72.3°, 147.5°, 252.3°$ or $327.5°$

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$5\sin^2 x = 6(1 - \cos x)$
$5\sin^2 x + 6\cos x - 6 = 0$
$5(1 - \cos^2 x) + 6\cos x - 6 = 0$
$5 - 5\cos^2 x + 6\cos x - 6 = 0$
$5\cos^2 x - 6\cos x + 1 = 0$
$(5\cos x - 1)(\cos x - 1) = 0$
So $\cos x = \frac{1}{5}$ or $\cos x = 1$
When $\cos x = \frac{1}{5}$, $x = 78.5°$
or $x = 360° - 78.5° = 281.5°$
When $\cos x = 1$, $x = 0°$ or $360°$
$x = 0°, 78.5°, 281.5°$ or $360°$

39 LHS $= \cos^2 x(\tan^2 x + 1)$

$$= \cos^2 x\left(\frac{\sin^2 x}{\cos^2 x} + 1\right)$$

$= \sin^2 x + \cos^2 x$
$=$ RHS

Challenge

1 a Finding points B and C using $y = 3x - 12$:
When $y = 0$, $x = 4$
When $x = 0$, $y = -12$
The point B is (4, 0) and
the point C is (0, −12).
Using Pythagoras' theorem to find the length of the square:
$BC = \sqrt{(0-4)^2 + (-12-0)^2} = \sqrt{160}$
Area of square $= \left(\sqrt{160}\right)^2 = 160$

b The point A is (−8, 4) and
the point D is (−12, −8).
The gradient of line $AD = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-8-4}{-12+8}$
$= \frac{-12}{-4}$
$= 3$
The equation of line AD is:
$y - y_1 = m(x - x_1)$
$y - 4 = 3(x + 8)$
$y = 3x + 28$
When $y = 0$, $x = -\frac{28}{3}$
The point S is $(-\frac{28}{3}, 0)$.

2 Rearranging $x^2 + y^2 + 8x - 10y = 59$:
$x^2 + 8x + y^2 - 10y = 59$
Completing the square:
$(x + 4)^2 - 16 + (y - 5)^2 - 25 = 59$
$(x + 4)^2 + (y - 5)^2 = 100$
Both circles have the same centre at (−4, 5). The radius of one circle is 8 and the other is 10, so
$(x + 4)^2 + (y - 5)^2 = 8^2$ lies completely inside $x^2 + y^2 + 8x - 10y = 59$.

3 LHS $= \binom{n}{k} + \binom{n}{k+1}$
$= \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!}$
$= \frac{n!(k+1)}{(k+1)!(n-k)!} + \frac{n!(n-k)}{(k+1)!(n-k)!}$
$= \frac{n!((k+1)+(n-k))}{(k+1)!(n-k)!}$
$= \frac{n!(n+1)}{(k+1)!(n-k)!}$
$= \frac{(n+1)!}{(k+1)!(n-k)!}$
$= \binom{n+1}{k+1}$
= RHS

4 $2\sin^3 x - \sin x + 1 = \cos^2 x$
$2\sin^3 x - \sin x + 1 = 1 - \sin^2 x$
$2\sin^3 x + \sin^2 x - \sin x = 0$
$\sin x(2\sin^2 x + \sin x - 1) = 0$
$\sin x(2\sin x - 1)(\sin x + 1) = 0$
So $\sin x = 0$, $\sin x = \frac{1}{2}$ or $\sin x = -1$
When $\sin x = 0$, $x = 0°, 180°$ or $360°$
When $\sin x = \frac{1}{2}$, $x = 30°$
or $x = 180° - 30° = 150°$
When $\sin x = -1$, $x = 270°$
So $x = 0°, 30°, 150°, 180°, 270°$ or $360°$