Integration 11A

1 a $\int\left(3\sec^2 x+\frac{5}{x}+\frac{2}{x^2}\right)dx$

$=\int\left(3\sec^2 x+\frac{5}{x}+2x^{-2}\right)dx$

$=3\tan x+5\ln|x|-\frac{2}{x}+c$

b $\int(5e^x-4\sin x+2x^3)dx$

$=5e^x+4\cos x+\frac{2x^4}{4}+c$

$=5e^x+4\cos x+\frac{x^4}{2}+c$

c $\int 2(\sin x-\cos x+x)dx$

$=\int(2\sin x-2\cos x+2x)dx$

$=-2\cos x-2\sin x+x^2+c$

d $\int\left(3\sec x\tan x-\frac{2}{x}\right)dx$

$=3\sec x-2\ln|x|+c$

e $\int\left(5e^x+4\cos x-\frac{2}{x^2}\right)dx$

$=\int(5e^x+4\cos x-2x^{-2})dx$

$=5e^x+4\sin x+\frac{2}{x}+c$

f $\int\left(\frac{1}{2x}+2\operatorname{cosec}^2 x\right)dx$

$=\int\left(\frac{1}{2}\times\frac{1}{x}+2\operatorname{cosec}^2 x\right)dx$

$=\frac{1}{2}\ln|x|-2\cot x+c$

g $\int\left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)dx$

$=\int\left(\frac{1}{x}+x^{-2}+x^{-3}\right)dx$

$=\ln|x|+\frac{x^{-1}}{-1}+\frac{x^{-2}}{-2}+c$

$=\ln|x|-\frac{1}{x}-\frac{1}{2x^2}+c$

h $\int(e^x+\sin x+\cos x)dx$

$=e^x-\cos x+\sin x+c$

i $\int(2\operatorname{cosec} x\cot x-\sec^2 x)dx$

$=-2\operatorname{cosec} x-\tan x+c$

j $\int\left(e^x+\frac{1}{x}-\operatorname{cosec}^2 x\right)dx$

$=e^x+\ln|x|+\cot x+c$

2 a $\int\left(\frac{1}{\cos^2 x}+\frac{1}{x^2}\right)dx$

$=\int(\sec^2 x+x^{-2})dx$

$=\tan x-\frac{1}{x}+c$

b $\int\left(\frac{\sin x}{\cos^2 x}+2e^x\right)dx$

$=\int(\tan x\sec x+2e^x)dx$

$=\sec x+2e^x+c$

c $\int\left(\frac{1+\cos x}{\sin^2 x}+\frac{1+x}{x^2}\right)dx$

$=\int(\operatorname{cosec}^2 x+\cot x\operatorname{cosec} x+x^{-2}+x^{-1})dx$

$=-\cot x-\operatorname{cosec} x-\frac{1}{x}+\ln|x|+c$

2 d $\int\left(\frac{1}{\sin^2 x}+\frac{1}{x}\right)dx$

$=\int\left(\operatorname{cosec}^2 x+\frac{1}{x}\right)dx$

$=-\cot x+\ln|x|+c$

2 e $\int \sin x(1+\sec^2 x)dx$

$= \int (\sin x + \sin x \sec^2 x)dx$

$= \int (\sin x + \tan x \sec x)dx$

$= -\cos x + \sec x + c$

f $\int \cos x(1+\text{cosec}^2 x)dx$

$= \int (\cos x + \cos x\,\text{cosec}^2 x)dx$

$= \int (\cos x + \cot x\,\text{cosec}\,x)dx$

$= \sin x - \text{cosec}\,x + c$

g $\int \text{cosec}^2 x(1+\tan^2 x)dx$

$= \int (\text{cosec}^2 x + \text{cosec}^2 x \tan^2 x)dx$

$= \int (\text{cosec}^2 x + \sec^2 x)dx$

$= -\cot x + \tan x + c$

h $\int \sec^2 x(1-\cot^2 x)dx$

$= \int (\sec^2 x - \sec^2 x \cot^2 x)dx$

$= \int (\sec^2 x - \text{cosec}^2 x)dx$

$= \tan x + \cot x + c$

i $\int \sec^2 x(1+e^x \cos^2 x)dx$

$= \int (\sec^2 x + e^x \cos^2 x \sec^2 x)dx$

$= \int (\sec^2 x + e^x)dx$

$= \tan x + e^x + c$

j $\int \left(\frac{1+\sin x}{\cos^2 x} + \cos^2 x \sec x\right)dx$

$= \int (\sec^2 x + \tan x \sec x + \cos x)dx$

$= \tan x + \sec x + \sin x + c$

3 a $\int_3^7 2e^x dx = \left[2e^x\right]_3^7 = 2e^7 - 2e^3$

b $\int_1^6 \frac{1+x}{x^3}dx = \int_1^6 \left(\frac{1}{x^3} + \frac{1}{x^2}\right)dx$

$= \left[-\frac{1}{2x^2} - \frac{1}{x}\right]_1^6 = \left(-\frac{1}{72} - \frac{1}{6}\right) - \left(-\frac{1}{2} - 1\right)$

$= -\frac{13}{72} + \frac{108}{72} = \frac{95}{72}$

c $\int_{\frac{\pi}{2}}^{\pi} -5\sin x dx = \left[5\cos x\right]_{\frac{\pi}{2}}^{\pi}$

$= 5\cos\pi - 5\cos\frac{\pi}{2} = -5 - 0 = -5$

d $\int_{-\frac{\pi}{4}}^{0} \sec x(\sec x + \tan x)dx$

$= \int_{-\frac{\pi}{4}}^{0} (\sec^2 x + \sec x \tan x)dx$

$= \left[\tan x + \sec x\right]_{-\frac{\pi}{4}}^{0} = (0+1) - \left(-1+\sqrt{2}\right)$

$= 2 - \sqrt{2}$

4 $\int_a^{2a} \frac{3x-1}{x}dx = \int_a^{2a} 3 - \frac{1}{x}dx = \left[3x - \ln|x|\right]_a^{2a}$

$= (6a - \ln 2a) - (3a - \ln a)$ (a is positive)

$= 3a - \ln 2a + \ln a$

$= 3a - (\ln 2 + \ln a) + \ln a$

$= 3a - \ln 2$

$= 3a + \ln\left(\frac{1}{2}\right) = 6 + \ln\left(\frac{1}{2}\right)$

so $a = 2$.

5 $\int_{\ln 1}^{\ln a} e^x + e^{-x}dx = \left[e^x - e^{-x}\right]_{\ln 1}^{\ln a}$

$= (e^{\ln a} - e^{-\ln a}) - (e^{\ln 1} - e^{-\ln 1})$

$= \left(a - \frac{1}{a}\right) - (1-1)$

So $a - \frac{1}{a} = \frac{48}{7}$

$7a^2 - 48a - 7 = 0$

$(7a+1)(a-7) = 0$

$a = 7$ since $a > 0$.

6 $\int_2^b (3e^x + 6e^{-2x})dx = \left[3e^x - 3e^{-2x}\right]_2^b$

$= 3((e^b - e^{-2b}) - (e^2 - e^{-4})) = 0$

$(e^b - e^{-2b}) - (e^2 - e^{-4}) = 0$

so $b = 2$.

7 a $f(x) = \frac{1}{8}x^{\frac{3}{2}} - \frac{4}{x} = 0$

$\frac{1}{8}x^{\frac{5}{2}} - 4 = 0$ since $x \neq 0$

$x^{\frac{5}{2}} = 32 \Rightarrow x = 4$

7 b $\int\left(\frac{1}{8}x^{\frac{3}{2}}-\frac{4}{x}\right)\mathrm{d}x=\frac{x^{\frac{5}{2}}}{20}-4\ln|x|+c$

c $\int_1^4 \mathrm{f}(x)\mathrm{d}x=\left[\frac{x^{\frac{5}{2}}}{20}-4\ln|x|\right]_1^4$

$=\left(\frac{32}{20}-4\ln 4\right)-\left(\frac{1}{20}-4\ln 1\right)$

$=\frac{31}{20}-4\ln 4$