Integration 11D

1 a $y = \ln|x^2+4|$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2+4} \times 2x$ (chain rule)

$\therefore \int \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln|x^2+4| + c$

b $y = \ln|e^{2x}+1|$

$\Rightarrow \frac{dy}{dx} = \frac{1}{e^{2x}+1} \times e^{2x} \times 2$ (chain rule)

$\therefore \int \frac{e^{2x}}{e^{2x}+1}\,dx = \frac{1}{2}\ln|e^{2x}+1| + c$

c $y = (x^2+4)^{-2}$

$\Rightarrow \frac{dy}{dx} = -2(x^2+4)^{-3} \times 2x$ (chain rule)

$\therefore \int \frac{x}{(x^2+4)^3}\,dx = -\frac{1}{4}(x^2+4)^{-2} + c$

or $-\frac{1}{4(x^2+4)^2} + c$

d $y = (e^{2x}+1)^{-2}$

$\Rightarrow \frac{dy}{dx} = -2(e^{2x}+1)^{-3} \times e^{2x} \times 2$ (chain rule)

$\therefore \int \frac{e^{2x}}{(e^{2x}+1)^3}\,dx = -\frac{1}{4}(e^{2x}+1)^{-2} + c$

or $-\frac{1}{4(e^{2x}+1)^2} + c$

e $y = \ln|3+\sin 2x|$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3+\sin 2x} \times \cos 2x \times 2$ (chain rule)

$\therefore \int \frac{\cos 2x}{3+\sin 2x}\,dx = \frac{1}{2}\ln|3+\sin 2x| + c$

f $y = (3+\cos 2x)^{-2}$

$\Rightarrow \frac{dy}{dx} = -2(3+\cos 2x)^{-3} \times (-\sin 2x) \times 2$
(chain rule)

$\therefore \int \frac{\sin 2x}{(3+\cos 2x)^3}\,dx = \frac{1}{4}(3+\cos 2x)^{-2} + c$

or $\frac{1}{4(3+\cos 2x)^2} + c$

g $y = e^{x^2}$

$\Rightarrow \frac{dy}{dx} = e^{x^2} \times 2x$ (chain rule)

$\therefore \int xe^{x^2}\,dx = \frac{1}{2}e^{x^2} + c$

h $y = (1+\sin 2x)^5$

$\Rightarrow \frac{dy}{dx} = 5(1+\sin 2x)^4 \times \cos 2x \times 2$
(chain rule)

$\therefore \int \cos 2x(1+\sin 2x)^4\,dx = \frac{1}{10}(1+\sin 2x)^5 + c$

i $y = \tan^3 x$

$\Rightarrow \frac{dy}{dx} = 3\tan^2 x \times \sec^2 x$ (chain rule)

$\therefore \int \sec^2 x \tan^2 x\,dx = \frac{1}{3}\tan^3 x + c$

j $\sec^2 x(1+\tan^2 x) = \sec^2 x + \sec^2 x \tan^2 x$

$\therefore \int \sec^2 x(1+\tan^2 x)\,dx$

$= \int \sec^2 x + \sec^2 x \tan^2 x\,dx$

$= \tan x + \frac{1}{3}\tan^3 x + c$

2 a $y=(x^2+2x+3)^5$

$$\Rightarrow \frac{dy}{dx}=5(x^2+2x+3)^4\times(2x+2)$$
$$=5(x^2+2x+3)^4\times 2(x+1)$$
$$\therefore \int (x+1)(x^2+2x+3)^4 dx$$
$$=\frac{1}{10}(x^2+2x+3)^5+c$$

b $y=\cot^2 2x$

$$\Rightarrow \frac{dy}{dx}=2\cot 2x\times(-\operatorname{cosec}^2 2x)\times 2$$
$$=-4\operatorname{cosec}^2 2x\cot 2x$$
$$\therefore \int \operatorname{cosec}^2 2x\cot 2x\, dx=-\frac{1}{4}\cot^2 2x+c$$

c $y=\sin^6 3x$

$$\Rightarrow \frac{dy}{dx}=6\sin^5 3x\times\cos 3x\times 3$$
$$\therefore \int \sin^5 3x\cos 3x\, dx=\frac{1}{18}\sin^6 3x+c$$

d $y=e^{\sin x}$

$$\Rightarrow \frac{dy}{dx}=e^{\sin x}\times\cos x$$
$$\therefore \int \cos x\, e^{\sin x}\, dx=e^{\sin x}+c$$

e $y=\ln|e^{2x}+3|$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{e^{2x}+3}\times e^{2x}\times 2$$
$$\therefore \int \frac{e^{2x}}{e^{2x}+3}dx=\frac{1}{2}\ln|e^{2x}+3|+c$$

f $y=(x^2+1)^{\frac{5}{2}}$

$$\Rightarrow \frac{dy}{dx}=\frac{5}{2}(x^2+1)^{\frac{3}{2}}\times 2x=5x(x^2+1)^{\frac{3}{2}}$$
$$\therefore \int x(x^2+1)^{\frac{3}{2}}dx=\frac{1}{5}(x^2+1)^{\frac{5}{2}}+c$$

g $y=(x^2+x+5)^{\frac{3}{2}}$

$$\Rightarrow \frac{dy}{dx}=\frac{3}{2}(x^2+x+5)^{\frac{1}{2}}\times(2x+1)$$
$$\therefore \int (2x+1)\sqrt{x^2+x+5}dx=\frac{2}{3}(x^2+x+5)^{\frac{3}{2}}+c$$

h $y=(x^2+x+5)^{\frac{1}{2}}$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}(x^2+x+5)^{-\frac{1}{2}}\times(2x+1)$$
$$=\frac{1}{2}\frac{(2x+1)}{\sqrt{x^2+x+5}}$$
$$\therefore \int \frac{2x+1}{\sqrt{x^2+x+5}}dx=2(x^2+x+5)^{\frac{1}{2}}+c$$

i $y=(\cos 2x+3)^{\frac{1}{2}}$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}(\cos 2x+3)^{-\frac{1}{2}}\times(-\sin 2x)\times 2$$
$$=-\frac{\sin 2x}{\sqrt{\cos 2x+3}}$$
$$=-\frac{2\sin x\cos x}{\sqrt{\cos 2x+3}}$$
$$\therefore \int \frac{\sin x\cos x}{\sqrt{\cos 2x+3}}dx=-\frac{1}{2}(\cos 2x+3)^{\frac{1}{2}}+c$$

j $y=\ln|\cos 2x+3|$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\cos 2x+3}\times(-\sin 2x)\times 2$$
$$=-\frac{2\sin 2x}{\cos 2x+3}$$
$$=-\frac{4\sin x\cos x}{\cos 2x+3}$$
$$\therefore \int \frac{\sin x\cos x}{\cos 2x+3}dx=-\frac{1}{4}\ln|\cos 2x+3|+c$$

3 a Let $I=\int_0^3(3x^2+10x)\sqrt{x^3+5x^2+9}\ dx$

Consider $y=(x^3+5x^2+9)^{\frac{3}{2}}$

$$\frac{dy}{dx}=\frac{3}{2}(3x^2+10x)(x^3+5x^2+9)^{\frac{1}{2}}$$

So $I=\left[\frac{2}{3}(x^3+5x^2+9)^{\frac{3}{2}}\right]_0^3$

$$=486-18=468$$

3 b Let $I = \int_{\frac{\pi}{9}}^{\frac{2\pi}{9}} \frac{6\sin 3x}{1-\cos 3x}\,dx$

Consider $y = \ln|1-\cos 3x|$

$\frac{dy}{dx} = \frac{3\sin 3x}{(1-\cos 3x)}$

So $I = \left[2\ln|1-\cos 3x|\right]_{\frac{\pi}{9}}^{\frac{2\pi}{9}}$

$= 2\left(\ln\frac{3}{2} - \ln\frac{1}{2}\right) = 2\ln 3$

c Let $I = \int_4^7 \frac{x}{x^2-1}\,dx$

Consider $y = \ln|x^2-1|$

$\frac{dy}{dx} = \frac{2x}{x^2-1}$

So $I = \left[\frac{1}{2}\ln|x^2-1|\right]_4^7$

$= \frac{1}{2}(\ln 48 - \ln 15)$

$= \frac{1}{2}\ln\frac{48}{15} = \frac{1}{2}\ln\frac{16}{5}$

d Let $I = \int_0^{\frac{\pi}{4}} \sec^2 x e^{4\tan x}\,dx$

Consider $y = e^{4\tan x}$

$\frac{dy}{dx} = 4\sec^2 x e^{4\tan x}$

So $I = \left[\frac{1}{4}e^{4\tan x}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{4}e^4 - \frac{1}{4}e^0 = \frac{1}{4}(e^4-1)$

4 Let $I = \int_0^k kx^2 e^{x^3}\,dx$

Consider $y = e^{x^3}$

$\frac{dy}{dx} = 3x^2 e^{x^3}$

So $I = \left[\frac{k}{3}e^{x^3}\right]_0^k$

$= \frac{k}{3}(e^{k^3}-1) = \frac{2}{3}(e^8-1)$

$k = 2$

5 Let $I = \int_0^{\theta} 4\sin 2x\cos^4 2x\,dx$

Consider $y = \cos^5 2x$

$\frac{dy}{dx} = -10\sin 2x\cos^4 2x$

So $I = \left[-\frac{2}{5}\cos^5 2x\right]_0^{\theta}$

$= \left(-\frac{2}{5}\cos^5 2\theta\right) + \frac{2}{5} = \frac{4}{5}$

$\cos^5 2\theta = -1 \Rightarrow \cos 2\theta = -1$

$2\theta = \pi \Rightarrow \theta = \frac{\pi}{2}$

6 a $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$

Consider $y = \ln|\sin x|$

$\frac{dy}{dx} = \frac{\cos x}{\sin x}$

So $\int \cot x\,dx = \ln|\sin x| + c$

b $\int \tan x\,dx \equiv \int \frac{\sin x}{\cos x}\,dx$

Consider $y = \ln|\cos x|$

$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$

So $\int \tan x\,dx = -\ln|\cos x| + c$

$\equiv \ln\left|\frac{1}{\cos x}\right| + c$

$\equiv \ln|\sec x| + c$