Integration 11I

1 a

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1	1.1260	1.2559	1.4142	1.6529

b i $\text{Area} \approx \frac{1}{2}h\left(y_0 + 2(y_1 + \ldots) + y_n\right)$

$= \frac{1}{2} \times \frac{\pi}{6}\left(1 + 1.6529 + 2 \times 1.2559\right) = 1.352$

ii $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{12}\left(1 + 1.6529 + 2(1.1260 + 1.2559 + 1.4142)\right) = 1.341$

2 a

θ	$-\frac{\pi}{5}$	$-\frac{\pi}{10}$	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$
y	0	0.7071	1	0.7071	0

b $R = \frac{1}{2} \times \frac{\pi}{10}\left(0 + 0 + 2(0.7071 + 1 + 0.7071)\right) = 0.758$

c The shape of the graph is concave, so the trapezium lines will underestimate the area.

d $\int_{-\frac{\pi}{5}}^{\frac{\pi}{5}} \cos\frac{5\theta}{2}\, d\theta = \left[\frac{2}{5}\sin\frac{5\theta}{2}\right]_{-\frac{\pi}{5}}^{\frac{\pi}{5}} = \frac{4}{5} = 0.8$

e $\text{Percentage error} = \frac{0.8 - 0.758}{0.8} \times 100\% = 5.25\%$

3 a

x	0	0.5	1	1.5	2
y	0.707	0.614	0.519	0.427	0.345

b Area using the trapezium rule:

$\approx \frac{1}{2}h\left(y_0 + 2(y_1 + \ldots) + y_n\right) = \frac{1}{4}\left(0.707 + 0.345 + 2(0.614 + 0.519 + 0.427)\right)$

= 1.04 to 2 decimal places

4 a

x	1	1.5	2	2.5	3
y	1	0.7973	1	1.4581	2.0986

b i $\text{Area} \approx \frac{1}{2} \times 1\left(1 + 2.0986 + 2\right) = 2.549$

ii $\text{Area} \approx \frac{1}{2} \times \frac{1}{2}\left(1 + 2.0986 + 2(0.7973 + 1 + 1.4581)\right) = 2.402$

c Increasing the number of values decreases the interval. This leads to an approximation more closely following the curve.

d $\int_1^3 \left((x-2)\ln x + 1\right) \mathrm{d}x = \int_1^3 (x-2)\ln x \ \mathrm{d}x + \int_1^3 \mathrm{d}x$

Let $u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\frac{\mathrm{d}v}{\mathrm{d}x} = x - 2 \Rightarrow v = \frac{(x-2)^2}{2}$

$I = \left[\frac{(x-2)^2 \ln x}{2}\right]_1^3 - \int_1^3 \frac{(x-2)^2}{2x} \mathrm{d}x + 2$

$= \frac{1}{2}\ln 3 - \int_1^3 \frac{x^2 - 4x + 4}{2x} \mathrm{d}x + 2$

$= \frac{1}{2}\ln 3 - \int_1^3 \left(\frac{x}{2} - 2 + \frac{2}{x}\right) \mathrm{d}x + 2$

$= \frac{1}{2}\ln 3 - \left[\frac{x^2}{4} - 2x + 2\ln x\right]_1^3 + 2$

$= \frac{1}{2}\ln 3 + 2 - \left(\frac{9}{4} - 6 + 2\ln 3\right) + \left(\frac{1}{4} - 2\right) = -\frac{3}{2}\ln 3 + 4$

5 a

x	0	0.5	1	1.5	2
y	0	0.6124	1	1.0607	0

b $\text{Area} \approx \frac{1}{2} \times 0.5\left(0 + 0 + 2(0.6124 + 1 + 1.0607)\right) = 1.337$

5 c $I = \int_0^2 x\sqrt{2-x}\ \mathrm{d}x$

Let $u = 2 - x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = -1$

$I = \int_2^0 -(2-u)u^{\frac{1}{2}}\ \mathrm{d}u = \int_2^0 u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\ \mathrm{d}u = \left[\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}}\right]_2^0$

$= 0 - \left(\frac{2}{5}\sqrt{32} - \frac{4}{3}\sqrt{8}\right) = \frac{8}{3}\sqrt{2} - \frac{8}{5}\sqrt{2} = \frac{16}{15}\sqrt{2} = \frac{2^{\frac{9}{2}}}{15}$

$p = \frac{16}{15}, q = \frac{1}{2}$

d $\frac{16}{15}\sqrt{2} = 1.509$

Percentage error $= \frac{1.509 - 1.337}{1.509} \times 100\% = 11.4\%$

6 a $y = \frac{4x-5}{(x-3)(2x+1)}$

$y = 0 \Rightarrow 4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

$A\left(\frac{5}{4}, 0\right)$

b

x	0	0.25	0.5	0.75	1	1.25
y	1.6667	0.9697	0.6	0.3556	0.1667	0

c Area $\approx \frac{1}{2} \times 0.25\left(1.6667 + 0 + 2(0.9697 + 0.6 + 0.3556 + 0.1667)\right) = 0.7313$

d $\frac{4x-5}{(x-3)(2x+1)} = \frac{A}{x-3} + \frac{B}{2x+1}$

$4x - 5 = A(2x+1) + B(x-3)$

Let $x = 3 : 7 = 7A \Rightarrow A = 1$

Let $x = -\frac{1}{2} : -7 = -\frac{7}{2}B \Rightarrow B = 2$

$y = \frac{1}{x-3} + \frac{2}{2x+1}$

$I = \int_0^{\frac{5}{4}}\left(\frac{1}{x-3} + \frac{2}{2x+1}\right)\ \mathrm{d}x = \left[\ln|x-3| + \ln|2x+1|\right]_0^{\frac{5}{4}}$

$= \ln\frac{7}{4} + \ln\frac{7}{2} - \ln 3 - \ln 1 = \ln 7 + \ln 7 - (\ln 4 + \ln 2 + \ln 3) = \ln\frac{49}{24}$

6 e $\ln\frac{49}{24} = 0.7137$

Percentage error $= \frac{0.7137 - 0.7313}{0.7137} \times 100\% = 2.5\%$

7 a

x	0	0.5	1	1.5	2	2.5	3
y	2.7183	4.1133	5.6522	7.3891	9.3565	11.5824	14.0940

b Area $\approx \frac{1}{2} \times 0.5\left(2.7183 + 14.0940 + 2(4.1133 + 5.6522 + 7.3891 + 9.3565 + 11.5824)\right) = 23.25$

c $I = \int_0^3 e^{\sqrt{2x+1}}\ dx$

Let $t = \sqrt{2x+1} \Rightarrow \frac{dt}{dx} = \frac{1}{\sqrt{2x+1}} = \frac{1}{t}$

$I = \int_1^{\sqrt{7}} te^t\ dt$

$a = 1, b = \sqrt{7}, k = 1$

d $I = \int_1^{\sqrt{7}} te^t\ dt$

Let $u = t \Rightarrow \frac{du}{dt} = 1$

$\frac{dv}{dt} = e^t \Rightarrow v = e^t$

$I = \left[te^t\right]_1^{\sqrt{7}} - \int_1^{\sqrt{7}} e^t\ dt = \sqrt{7}e^{\sqrt{7}} - e - e^{\sqrt{7}} + e = \left(\sqrt{7} - 1\right)e^{\sqrt{7}} = 23.20$