Vectors 12D

1 a $\mathbf{R} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + 7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} - 5\mathbf{i} - 3\mathbf{j}$
$= (5\mathbf{i} - \mathbf{j} + 4\mathbf{k})\text{ N}$

b $|\mathbf{R}| = \sqrt{5^2 + 1 + 4^2} = \sqrt{42}\text{ N}$

2 $|\mathbf{a}| = \sqrt{4^2 + 2^2 + 3^2} = \sqrt{29}\,\text{m s}^{-2}$

Using $s = ut + \frac{1}{2}at^2$:

$s = \frac{1}{2}\sqrt{29} \times 4 = 2\sqrt{29}\text{ m}$

3 a $\mathbf{F} = m\mathbf{a} \Rightarrow 2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k} = 4\mathbf{a}$
$\mathbf{a} = \left(\frac{1}{2}\mathbf{i} - \frac{5}{4}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)\text{m s}^{-2}$

b $|\mathbf{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5}{4}\right)^2 + \left(\frac{3}{4}\right)^2}$
$= 1.54\,\text{m s}^{-2}$

4 $\mathbf{F}_1 + \mathbf{F}_2 = m\mathbf{a} \Rightarrow 7\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mathbf{F}_2 = 6(2\mathbf{i} - \mathbf{k})$
$\mathbf{F}_2 = (12\mathbf{i} - 7\mathbf{i} - 3\mathbf{j} - 6\mathbf{k} - \mathbf{k})$
$= (5\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})\text{ N}$

5 a Particle is in equilibrium
$\Rightarrow \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$
$(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + (-\mathbf{i} + 3\mathbf{j} + b\mathbf{k}) + (a\mathbf{j} - 2\mathbf{k}) = \mathbf{0}$

Comparing coefficients of $\mathbf{j}$:
$-1 + 3 + a = 0 \Rightarrow a = -2$

Comparing coefficients of $\mathbf{k}$:
$-2 + b - 2 = 0 \Rightarrow b = 4$

b $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_3 = \mathbf{i} + (a - 1)\mathbf{j} - 4\mathbf{k}$
$= (\mathbf{i} - 3\mathbf{j} - 4\mathbf{k})\text{ N}$

c $\mathbf{F} = m\mathbf{a} \Rightarrow \mathbf{i} - 3\mathbf{j} - 4\mathbf{k} = 2\mathbf{a}$
$\mathbf{a} = \left(\frac{1}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} - 2\mathbf{k}\right)\text{m s}^{-2}$

d $|\mathbf{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + 2^2}$
$= \frac{1}{2}\sqrt{26}\,\text{m s}^{-2}$

e $\cos\theta_j = \frac{-\frac{3}{2}}{\frac{\sqrt{26}}{2}} = \frac{-3}{\sqrt{26}}$
$\theta_j = 126°$

This question has been removed from the latest edition of the book.

6 a Gravitational force downwards
$= 1200 \times 9.8 = 11760\text{ N}$

Total force on aeroplane
$= \mathbf{T} + \mathbf{L} + \mathbf{F} - 11760\mathbf{k}$
$= (1900\mathbf{i} - 1300\mathbf{j} - 460\mathbf{k})\text{ N}$

$\mathbf{F} = m\mathbf{a} \Rightarrow 1900\mathbf{i} - 1300\mathbf{j} - 460\mathbf{k} = 1200\mathbf{a}$
$\mathbf{a} = \left(\frac{19}{12}\mathbf{i} - \frac{13}{12}\mathbf{j} - \frac{4.6}{12}\mathbf{k}\right)\text{m s}^{-2}$
$|\mathbf{a}| = \sqrt{\left(\frac{19}{12}\right)^2 + \left(\frac{13}{12}\right)^2 + \left(\frac{4.6}{12}\right)^2}$
$= 1.96\text{ ms}^{-2}$

b As the aeroplane is initially in level flight and the acceleration in the vertical direction is $-460\,\text{m s}^{-2}$, the aeroplane must be descending.

$\cos\theta_k = \frac{-\frac{4.6}{12}}{1.96} = -0.1956...$
$\theta_k = 101.3°$