Algebraic methods 1B

1 a $\frac{a}{d} \times \frac{a}{c} = \frac{a \times a}{d \times c}$

$= \frac{a^2}{cd}$

b $\frac{a^{\cancel{2}}}{\cancel{c}_1} \times \frac{\cancel{c}^1}{\cancel{a}_1} = \frac{a \times 1}{1 \times 1}$

$= a$

c $\frac{\cancel{2}^1}{\cancel{x}_1} \times \frac{x^1}{\cancel{4}_2} = \frac{1 \times 1}{1 \times 2}$

$= \frac{1}{2}$

d $\frac{3}{x} \div \frac{6}{x} = \frac{\cancel{3}^1}{\cancel{x}_1} \times \frac{\cancel{x}^1}{\cancel{6}_2}$

$= \frac{1 \times 1}{1 \times 2}$

$= \frac{1}{2}$

e $\frac{4}{xy} \div \frac{x}{y} = \frac{4}{\cancel{xy}_1} \times \frac{\cancel{y}^1}{x}$

$= \frac{4 \times 1}{x \times x}$

$= \frac{4}{x^2}$

f $\frac{2r^2}{5} \div \frac{4}{r^3} = \frac{1}{\cancel{2}} \frac{\cancel{2}r^2}{5} \times \frac{r^3}{\cancel{4}_2}$

$= \frac{r^5}{10}$

2 a $(x+2)\times\dfrac{1}{x^2-4}=\dfrac{\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

$=\dfrac{1}{1\times(x-2)}$

$=\dfrac{1}{x-2}$

b $\dfrac{1}{a^2+6a+9}\times\dfrac{a^2-9}{2}$

$=\dfrac{1}{\cancel{(a+3)}(a+3)}\times\dfrac{\cancel{(a+3)}(a-3)}{2}$

$=\dfrac{a-3}{2(a+3)}$

c $\dfrac{x^2-3x}{y^2+y}\times\dfrac{y+1}{x}$

$=\dfrac{x^{1}(x-3)}{y\cancel{(y+1)}_{1}}\times\dfrac{\cancel{(y+1)}^{1}}{\cancel{x}_{1}}$

$=\dfrac{x-3}{y}$

d $\dfrac{y}{y+3}\div\dfrac{y^2}{y^2+4y+3}$

$=\dfrac{y}{y+3}\times\dfrac{y^2+4y+3}{y^2}$

$=\dfrac{\cancel{y}}{\cancel{y+3}}\times\dfrac{(y+1)\cancel{(y+3)}}{\cancel{y^2}}$

$=\dfrac{y+1}{y}$

2 e $\dfrac{x^2}{3} \div \dfrac{2x^3 - 6x^2}{x^2 - 3x} = \dfrac{x^2}{3} \times \dfrac{x^2 - 3x}{2x^3 - 6x^2}$

$$= \frac{\cancel{x^2}}{3} \times \frac{x\cancel{(x-3)}^{1}}{2\cancel{x^2}\,\cancel{(x-3)}_{1}}$$

$$= \frac{1 \times x}{3 \times 2}$$

$$= \frac{x}{6}$$

f $\dfrac{4x^2 - 25}{4x - 10} \div \dfrac{2x + 5}{8}$

$$= \frac{4x^2 - 25}{4x - 10} \times \frac{8}{(2x + 5)}$$

$$= \frac{\cancel{(2x+5)}^{1}\,\cancel{(2x-5)}^{1}}{2\cancel{(2x-5)}_{1}} \times \frac{8}{\cancel{(2x+5)}_{1}}$$

$$= \frac{1 \times 8}{2 \times 1}$$

$$= 4$$

g $\dfrac{x + 3}{x^2 + 10x + 25} \times \dfrac{x^2 + 5x}{x^2 + 3x}$

$$= \frac{\cancel{x+3}^{1}}{\cancel{(x+5)}_{1}(x+5)} \times \frac{x^{1}\,\cancel{(x+5)}^{1}}{\cancel{x}_{1}\,\cancel{(x+3)}_{1}}$$

$$= \frac{1}{x + 5}$$

h $\dfrac{3y^2 + 4y - 4}{10} \div \dfrac{3y + 6}{15}$

$$= \frac{3y^2 + 4y - 4}{10} \times \frac{15}{3y + 6}$$

$$= \frac{(3y-2)\,\cancel{(y+2)}^{1}}{\cancel{10}_{2}} \times \frac{\cancel{15}^{\cancel{3}}}{\cancel{3}\,\cancel{(y+2)}_{1}}$$

$$= \frac{3y - 2}{2}$$

2 i $\dfrac{x^2+2xy+y^2}{2}\times\dfrac{4}{(x-y)^2}$

$=\dfrac{(x+y)^2}{\cancel{2}_1}\times\dfrac{\cancel{4}^2}{(x-y)^2}$

$=\dfrac{2(x+y)^2}{(x-y)^2}$

3 $\dfrac{x^2-64}{x^2-36}\div\dfrac{64-x^2}{x^2-36}$

$=\dfrac{x^2-64}{x^2-36}\times\dfrac{x^2-36}{64-x^2}$

$=\dfrac{\cancel{(x+8)}(x-8)}{\cancel{(x+6)}\cancel{(x-6)}}\times\dfrac{\cancel{(x+6)}\cancel{(x-6)}}{\cancel{(8+x)}(8-x)}$

$=\dfrac{(x-8)}{(8-x)}$

$=\dfrac{(x-8)}{-(x-8)}$

$=-1$

4 $\dfrac{2x^2-11x-40}{x^2-4x-32}\times\dfrac{x^2+8x+16}{6x^2-3x-45}\div\dfrac{8x^2+20x-48}{10x^2-45x+45}$

$=\dfrac{2x^2-11x-40}{x^2-4x-32}\times\dfrac{x^2+8x+16}{6x^2-3x-45}\times\dfrac{10x^2-45x+45}{8x^2+20x-48}$

$=\dfrac{\cancel{(2x+5)}\cancel{(x-8)}}{\cancel{(x+4)}\cancel{(x-8)}}\times\dfrac{\cancel{(x+4)}\cancel{(x+4)}}{3\cancel{(2x+5)}\cancel{(x-3)}}\times\dfrac{5\cancel{(2x-3)}\cancel{(x-3)}}{4\cancel{(2x-3)}\cancel{(x+4)}}$

$=1\times\dfrac{1}{3}\times\dfrac{5}{4}$

$=\dfrac{5}{12}$

$a = 5$, $b = 12$

5 a $\dfrac{x^2+2x-24}{2x^2+10x}\times\dfrac{x^2-3x}{x^2+3x-18}$

$=\dfrac{\cancel{(x+6)}(x-4)}{2\cancel{x}(x+5)}\times\dfrac{\cancel{x}\cancel{(x-3)}}{\cancel{(x+6)}\cancel{(x-3)}}$

$=\dfrac{(x-4)}{2(x+5)}$

$=\dfrac{x-4}{2x+10}$

5 b $\ln\left(\left(x^2+2x-24\right)\left(x^2-3x\right)\right)$
$-\ln\left(\left(2x^2+10x\right)\left(x^2+3x-18\right)\right)=2$

$$\ln\left(\frac{\left(x^2+2x-24\right)\left(x^2-3x\right)}{\left(2x^2+10x\right)\left(x^2+3x-18\right)}\right)=2$$

$$\ln\left(\frac{\cancel{(x+6)}\,(x-4)\,\cancel{x}\,\cancel{(x-3)}}{\cancel{x}\,(2x+10)\,\cancel{(x+6)}\,\cancel{(x-3)}}\right)=2$$

$$\ln\left(\frac{x-4}{2x+10}\right)=2$$

$$\frac{x-4}{2x+10}=\mathrm{e}^2$$

$$x-4=2x\mathrm{e}^2+10\mathrm{e}^2$$

$$x\left(1-2\mathrm{e}^2\right)=10\mathrm{e}^2+4$$

$$x=\frac{10\mathrm{e}^2+4}{1-2\mathrm{e}^2}$$

6 a $\mathrm{f}(x)=\dfrac{2x^2-3x-2}{6x-8}\div\dfrac{x-2}{3x^2+14x-24}$

$$=\frac{2x^2-3x-2}{6x-8}\times\frac{3x^2+14x-24}{x-2}$$

$$=\frac{(2x+1)\,\cancel{(x-2)}}{2\,\cancel{(3x-4)}}\times\frac{\cancel{(3x-4)}\,(x+6)}{\cancel{x-2}}$$

$$=\frac{(2x+1)(x+6)}{2}$$

$$=\frac{2x^2+13x+6}{2}$$

b $\mathrm{f}(x)=x^2+\dfrac{13}{2}x+3$

$$\mathrm{f}'(x)=2x+\frac{13}{2}$$

$$\mathrm{f}'(4)=2\times4+\frac{13}{2}=\frac{29}{2}$$