Algebraic Methods 1C

1 a $\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}$
$=\frac{7}{12}$

b $\frac{3}{4}-\frac{2}{5}=\frac{15}{20}-\frac{8}{20}$
$=\frac{7}{20}$

c $\frac{1}{p}+\frac{1}{q}=\frac{q}{pq}+\frac{p}{pq}$
$=\frac{p+q}{pq}$

d $\frac{3}{4x}+\frac{1}{8x}=\frac{6}{8x}+\frac{1}{8x}$
$=\frac{7}{8x}$

e $\frac{3}{x^2}-\frac{1}{x}=\frac{3}{x^2}-\frac{x}{x^2}$
$=\frac{3-x}{x^2}$

f $\frac{a}{5b}-\frac{3}{2b}=\frac{2a}{10b}-\frac{15}{10b}$
$=\frac{2a-15}{10b}$

2 a $\frac{3}{x}-\frac{2}{x+1}=\frac{3(x+1)}{x(x+1)}-\frac{2x}{x(x+1)}$
$=\frac{3x+3-2x}{x(x+1)}$
$=\frac{x+3}{x(x+1)}$

b $\frac{2}{x-1}-\frac{3}{x+2}$
$=\frac{2(x+2)}{(x-1)(x+2)}-\frac{3(x-1)}{(x-1)(x+2)}$
$=\frac{2(x+2)-3(x-1)}{(x-1)(x+2)}$
$=\frac{2x+4-3x+3}{(x-1)(x+2)}$
$=\frac{-x+7}{(x-1)(x+2)}$

c $\frac{4}{2x+1}+\frac{2}{x-1}$
$=\frac{4(x-1)}{(2x+1)(x-1)}+\frac{2(2x+1)}{(2x+1)(x-1)}$
$=\frac{4(x-1)+2(2x+1)}{(2x+1)(x-1)}$
$=\frac{4x-4+4x+2}{(2x+1)(x-1)}$
$=\frac{8x-2}{(2x+1)(x-1)}$

d $\frac{1}{3}(x+2)-\frac{1}{2}(x+3)$
$=\frac{2}{6}(x+2)-\frac{3}{6}(x+3)$
$=\frac{2(x+2)-3(x+3)}{6}$
$=\frac{2x+4-3x-9}{6}$
$=\frac{-x-5}{6}$

e $\frac{3x}{(x+4)^2}-\frac{1}{x+4}$
$=\frac{3x}{(x+4)^2}-\frac{x+4}{(x+4)^2}$
$=\frac{3x-x-4}{(x+4)^2}$
$=\frac{2x-4}{(x+4)^2}$

2 f $\dfrac{5}{2(x+3)}+\dfrac{4}{3(x-1)}$

$=\dfrac{15(x-1)}{6(x+3)(x-1)}+\dfrac{8(x+3)}{6(x+3)(x-1)}$

$=\dfrac{15(x-1)+8(x+3)}{6(x+3)(x-1)}$

$=\dfrac{15x-15+8x+24}{6(x+3)(x-1)}$

$=\dfrac{23x+9}{6(x+3)(x-1)}$

3 a $\dfrac{2}{x^2+2x+1}+\dfrac{1}{x+1}$

$=\dfrac{2}{(x+1)^2}+\dfrac{1}{x+1}$

$=\dfrac{2}{(x+1)^2}+\dfrac{x+1}{(x+1)^2}$

$=\dfrac{2+x+1}{(x+1)^2}$

$=\dfrac{x+3}{(x+1)^2}$

b $\dfrac{7}{x^2-4}+\dfrac{3}{x+2}$

$=\dfrac{7}{(x+2)(x-2)}+\dfrac{3}{x+2}$

$=\dfrac{7}{(x+2)(x-2)}+\dfrac{3(x-2)}{(x+2)(x-2)}$

$=\dfrac{7+3(x-2)}{(x+2)(x-2)}$

$=\dfrac{7+3x-6}{(x+2)(x-2)}$

$=\dfrac{3x+1}{(x+2)(x-2)}$

c $\dfrac{2}{x^2+6x+9}-\dfrac{3}{x^2+4x+3}$

$=\dfrac{2}{(x+3)^2}-\dfrac{3}{(x+3)(x+1)}$

$=\dfrac{2(x+1)}{(x+3)^2(x+1)}-\dfrac{3(x+3)}{(x+3)^2(x+1)}$

$=\dfrac{2(x+1)-3(x+3)}{(x+3)^2(x+1)}$

$=\dfrac{2x+2-3x-9}{(x+3)^2(x+1)}$

$=\dfrac{-x-7}{(x+3)^2(x+1)}$

d $\dfrac{2}{y^2-x^2}+\dfrac{3}{y-x}$

$=\dfrac{2}{(y+x)(y-x)}+\dfrac{3}{y-x}$

$=\dfrac{2}{(y+x)(y-x)}+\dfrac{3(y+x)}{(y+x)(y-x)}$

$=\dfrac{2+3(y+x)}{(y+x)(y-x)}$

$=\dfrac{3x+3y+2}{(y+x)(y-x)}$

e $\dfrac{3}{x^2+3x+2}-\dfrac{1}{x^2+4x+4}$

$=\dfrac{3}{(x+1)(x+2)}-\dfrac{1}{(x+2)^2}$

$=\dfrac{3(x+2)}{(x+1)(x+2)^2}-\dfrac{(x+1)}{(x+1)(x+2)^2}$

$=\dfrac{3(x+2)-(x+1)}{(x+1)(x+2)^2}$

$=\dfrac{3x+6-x-1}{(x+1)(x+2)^2}$

$=\dfrac{2x+5}{(x+1)(x+2)^2}$

3 f $\dfrac{x+2}{x^2-x-12}-\dfrac{x+1}{x^2+5x+6}$

$=\dfrac{x+2}{(x-4)(x+3)}-\dfrac{x+1}{(x+2)(x+3)}$

$=\dfrac{(x+2)^2}{(x-4)(x+2)(x+3)}-\dfrac{(x+1)(x-4)}{(x-4)(x+2)(x+3)}$

$=\dfrac{(x+2)^2-(x+1)(x-4)}{(x-4)(x+2)(x+3)}$

$=\dfrac{x^2+4x+4-x^2+3x+4}{(x-4)(x+2)(x+3)}$

$=\dfrac{7x+8}{(x-4)(x+2)(x+3)}$

4 $\dfrac{6x+1}{x^2+2x-15}-\dfrac{4}{x-3}$

$=\dfrac{6x+1}{(x+5)(x-3)}-\dfrac{4}{x-3}$

$=\dfrac{6x+1}{(x+5)(x-3)}-\dfrac{4(x+5)}{(x+5)(x-3)}$

$=\dfrac{6x+1-4(x+5)}{(x+5)(x-3)}$

$=\dfrac{6x+1-4x-20}{(x+5)(x-3)}$

$=\dfrac{2x-19}{(x+5)(x-3)}$

5 a $\dfrac{3}{x}+\dfrac{2}{x+1}+\dfrac{1}{x+2}$

$=\dfrac{3(x+1)(x+2)}{x(x+1)(x+2)}+\dfrac{2x(x+2)}{x(x+1)(x+2)}+\dfrac{x(x+1)}{x(x+1)(x+2)}$

$=\dfrac{3(x+1)(x+2)+2x(x+2)+x(x+1)}{x(x+1)(x+2)}$

$=\dfrac{3x^2+9x+6+2x^2+4x+x^2+x}{x(x+1)(x+2)}$

$=\dfrac{6x^2+14x+6}{x(x+1)(x+2)}$

b $\dfrac{4}{3x}-\dfrac{2}{x-2}+\dfrac{1}{2x+1}$

$=\dfrac{4(x-2)(2x+1)}{3x(x-2)(2x+1)}-\dfrac{6x(2x+1)}{3x(x-2)(2x+1)}+\dfrac{3x(x-2)}{3x(x-2)(2x+1)}$

$=\dfrac{4(x-2)(2x+1)-6x(2x+1)+3x(x-2)}{3x(x-2)(2x+1)}$

$=\dfrac{8x^2-12x-8-12x^2-6x+3x^2-6x}{3x(x-2)(2x+1)}$

$=\dfrac{-x^2-24x-8}{3x(x-2)(2x+1)}$

c $\dfrac{3}{x-1}+\dfrac{2}{x+1}+\dfrac{4}{x-3}$

$=\dfrac{3(x+1)(x-3)}{(x-1)(x+1)(x-3)}+\dfrac{2(x-1)(x-3)}{(x-1)(x+1)(x-3)}+\dfrac{4(x-1)(x+1)}{(x-1)(x+1)(x-3)}$

$=\dfrac{3(x+1)(x-3)+2(x-1)(x-3)+4(x-1)(x+1)}{(x-1)(x+1)(x-3)}$

$=\dfrac{3x^2-6x-9+2x^2-8x+6+4x^2-4}{(x-1)(x+1)(x-3)}$

$=\dfrac{9x^2-14x-7}{(x-1)(x+1)(x-3)}$

6 $\dfrac{4(2x-1)}{36x^2-1}+\dfrac{7}{6x-1}$

$=\dfrac{4(2x-1)}{(6x-1)(6x+1)}+\dfrac{7}{6x-1}$

$=\dfrac{4(2x-1)}{(6x-1)(6x+1)}+\dfrac{7(6x+1)}{(6x-1)(6x+1)}$

$=\dfrac{4(2x-1)+7(6x+1)}{(6x-1)(6x+1)}$

$=\dfrac{8x-4+42x+7}{(6x-1)(6x+1)}$

$=\dfrac{50x+3}{(6x-1)(6x+1)}$

7 a $g(x) = x + \dfrac{6}{x+2} + \dfrac{36}{x^2-2x-8}$

$= x + \dfrac{6}{x+2} + \dfrac{36}{(x-4)(x+2)}$

$= \dfrac{x(x+2)(x-4)}{(x+2)(x-4)} + \dfrac{6(x-4)}{(x+2)(x-4)} + \dfrac{36}{(x+2)(x-4)}$

$= \dfrac{x(x+2)(x-4)+6(x-4)+36}{(x+2)(x-4)}$

$= \dfrac{x^3-2x^2-8x+6x-24+36}{(x+2)(x-4)}$

$= \dfrac{x^3-2x^2-2x+12}{(x+2)(x-4)}$

b Using the factor theorem,

$(-2)^3 - 2(-2)^2 - 2(-2) + 12 = 0$

So $(x+2)$ is a factor of

$x^3-2x^2-2x+12$

Hence, you can write

$x^3-2x^2-2x+12 = (x+2)\times p(x)$

for some quadratic polynomial $p(x)$.

You can find $p(x)$ by long division:

$$\begin{array}{r} x^2-4x+6 \\ x+2\overline{)\,x^3-2x^2-2x+12} \\ \underline{x^3+2x^2} \\ -4x^2-2x \\ \underline{-4x^2-8x} \\ 6x+12 \\ \underline{6x+12} \\ 0 \end{array}$$

Hence, $p(x) = x^2-4x+6$ and so

$g(x) = \dfrac{(x+2)(x^2-4x+6)}{(x+2)(x-4)}$

$= \dfrac{x^2-4x+6}{x-4}$