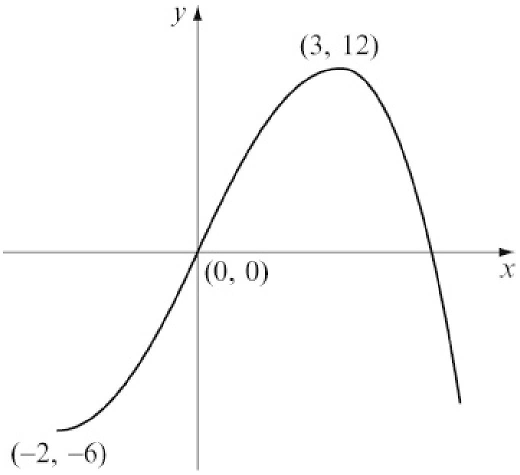
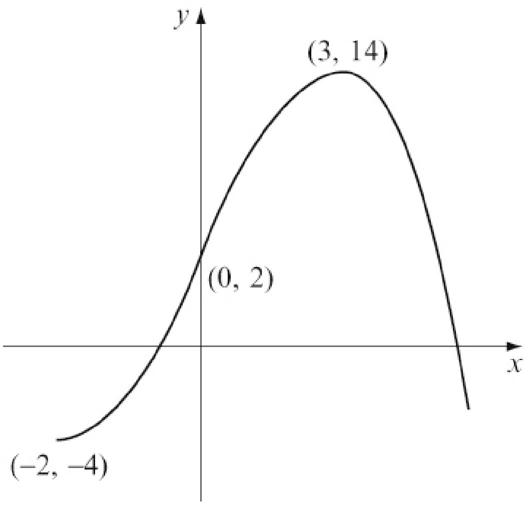
Functions and graphs 2F

1 a $y = 3\text{f}(x)$

Vertical stretch, scale factor 3.

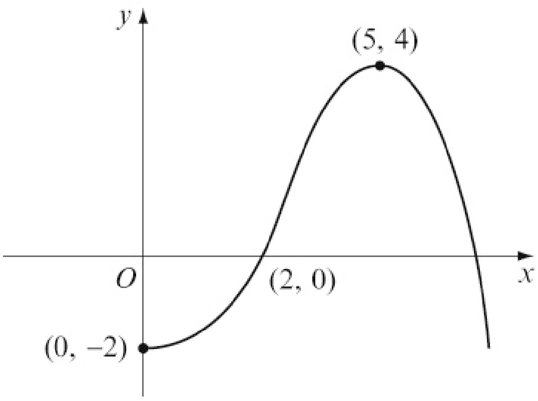


$y = 3\text{f}(x) + 2$. Vertical translation of +2.



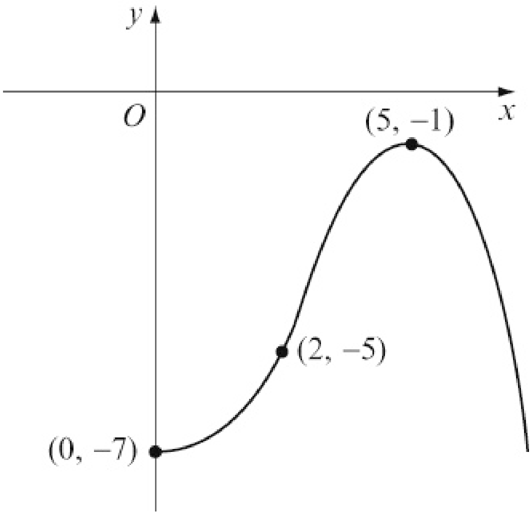
b $y = \text{f}(x-2)$.

Horizontal translation of $+2$.



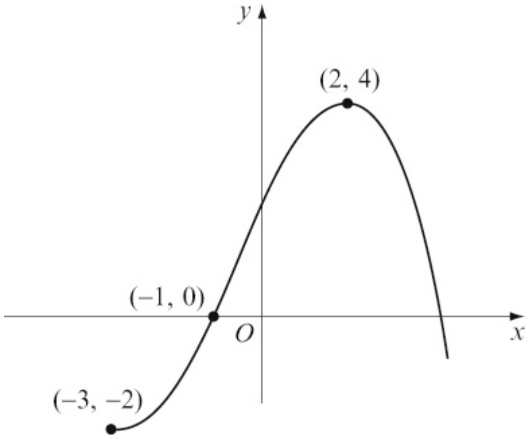
$y = \text{f}(x-2) - 5$.

Vertical translation of -5.



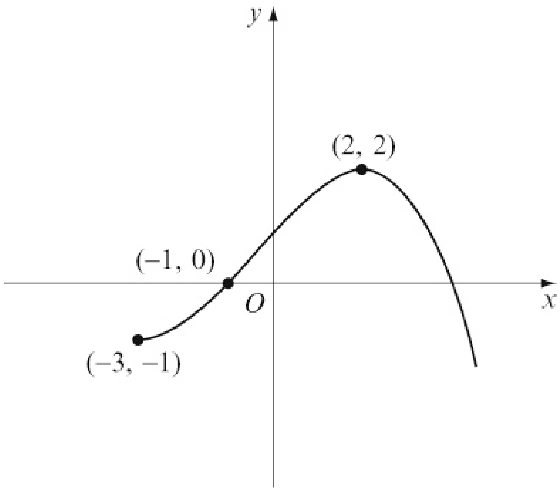
1 c $y = \mathrm{f}(x+1)$

Horizontal translation of -1.



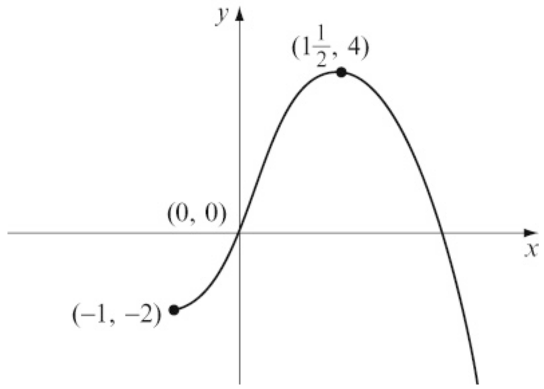
$y = \frac{1}{2}\mathrm{f}(x+1)$

Vertical stretch, scale factor $\frac{1}{2}$



d $y = \mathrm{f}(2x)$

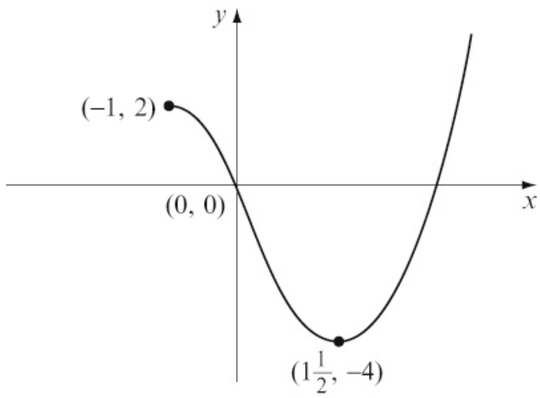
Horizontal stretch, scale factor $\frac{1}{2}$



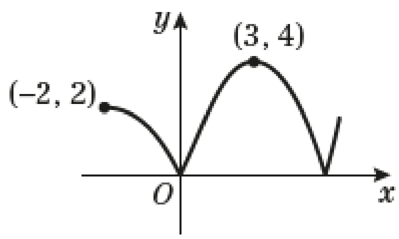
$y = -\mathrm{f}(2x)$

Reflection in the x-axis.

(or Vertical stretch, scale factor -1).



e $y = |\mathrm{f}(x)|$. Reflect, in the x-axis, the parts of the graph that lie below the x-axis.

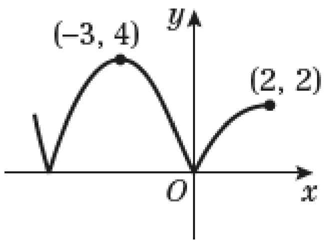


1 f $y = \mathrm{f}(-x)$.

Reflection in the y-axis.

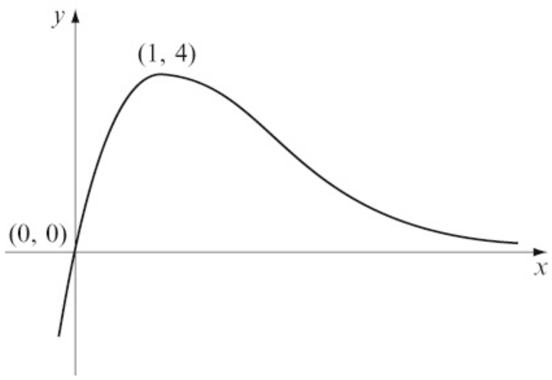
$y = |\mathrm{f}(-x)|$.

Reflect, in the x-axis, the parts of the graph that lie below the x-axis.



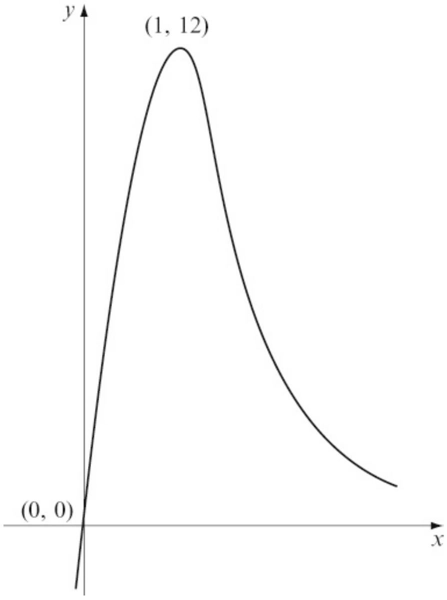
2 a $y = \mathrm{f}(x-2)$

Horizontal translation of $+2$



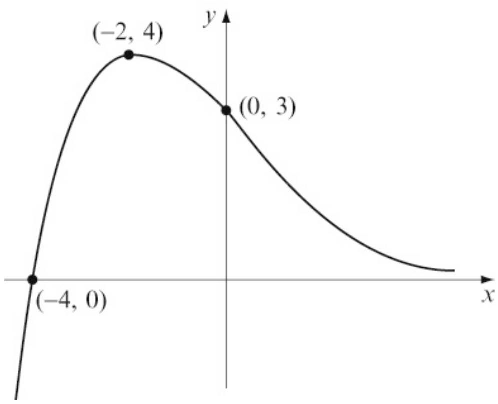
$y = 3\mathrm{f}(x-2)$

Vertical stretch, scale factor 3.



b $y = \mathrm{f}\left(\frac{1}{2}x\right)$

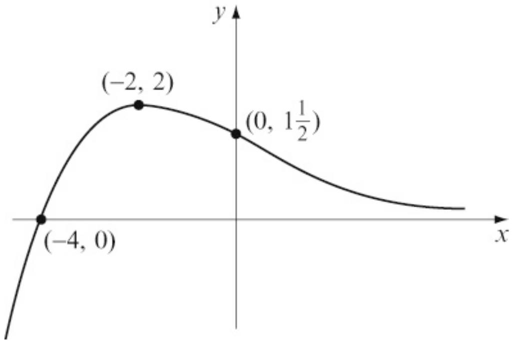
Horizontal stretch, scale factor 2.



2 b (continued)

$y = \frac{1}{2}f\left(\frac{1}{2}x\right)$

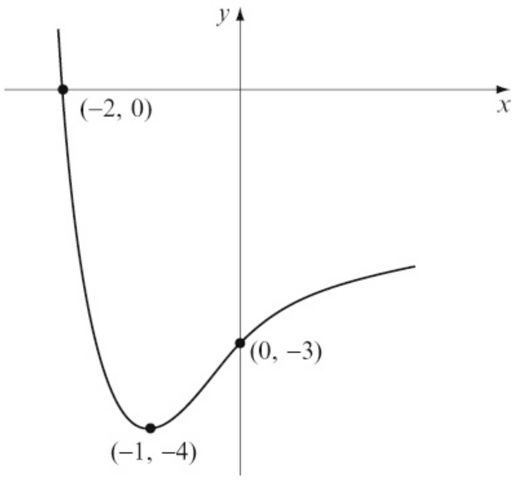
Vertical stretch, scale factor $\frac{1}{2}$



c $y = -f(x)$

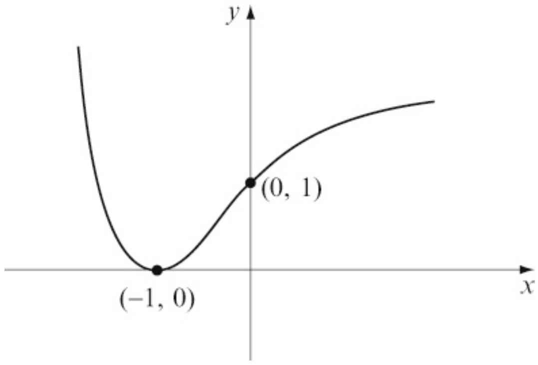
Reflection in the x-axis.

(Or vertical stretch, scale factor -1).



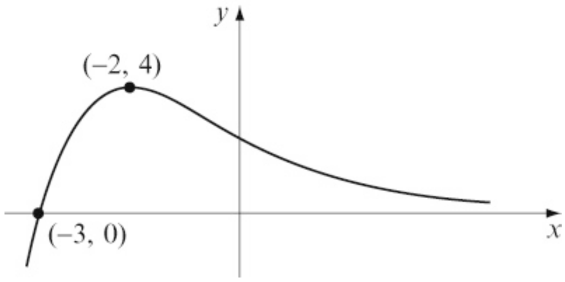
$y = -f(x) + 4$

Vertical translation of $+4$.



d $y = f(x+1)$

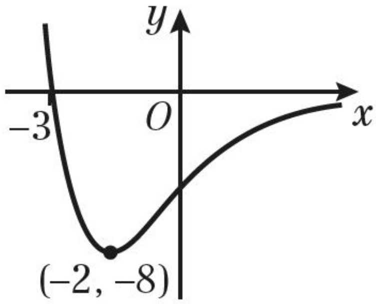
Horizontal translation of -1.



$y = -2f(x+1)$

Reflection in the x-axis,

and vertical stretch, scale factor 2.



2 e $y = \mathrm{f}(|x|)$ can be written

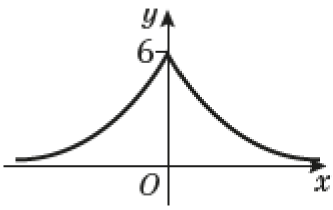
$$y = \begin{cases} \mathrm{f}(x), & x \geq 0 \\ \mathrm{f}(-x), & x < 0 \end{cases}$$

$y = \mathrm{f}(-x)$ is a reflection of $y = \mathrm{f}(x)$ in the y-axis.

Hence, $y = \mathrm{f}(|x|)$ is the following:

$y = 2\mathrm{f}(|x|)$

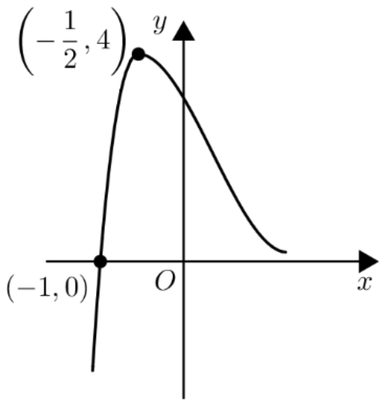
Vertical stretch, scale factor 2.



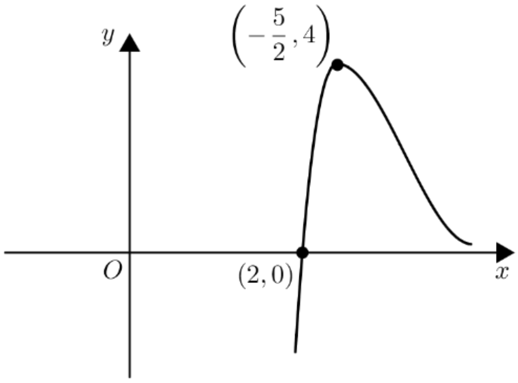
2 f $y = \mathrm{f}(2x - 6)$ can be written as

$y = \mathrm{f}(2(x - 3))$

$y = \mathrm{f}(2x)$: Horizontal stretch, scale factor $\frac{1}{2}$

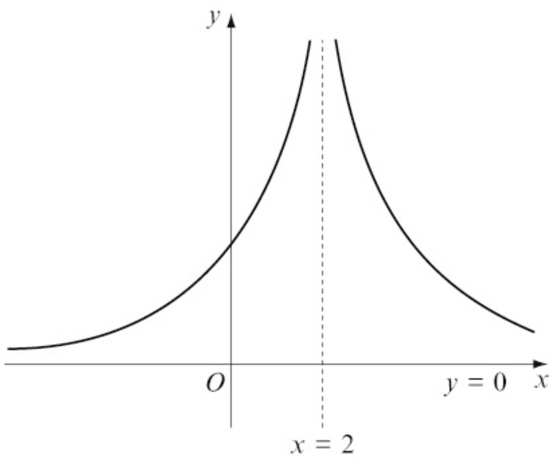


$y = \mathrm{f}(2(x - 3))$: Horizontal translation of $+3$



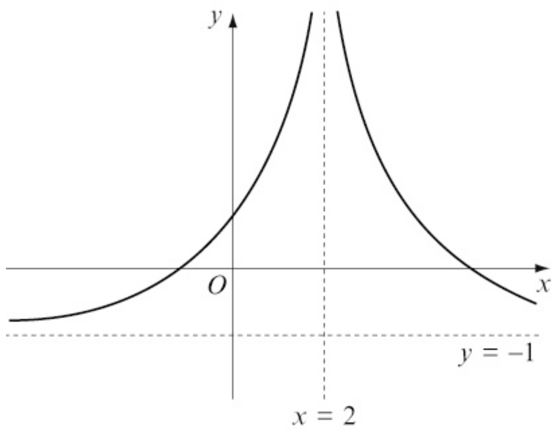
3 a $y = 3\mathrm{f}(x)$

Vertical stretch, scale factor 3.



$y = 3\mathrm{f}(x) - 1$

Vertical translation of -1.

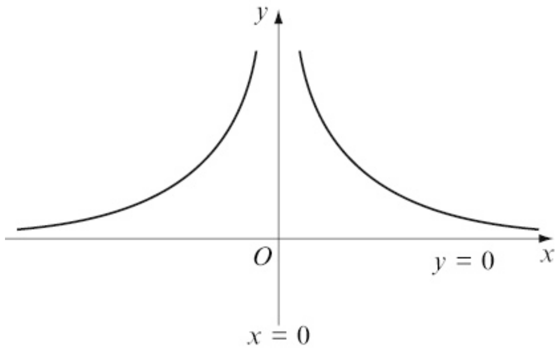


Asymptotes: $x = 2$, $y = -1$

A: (0, 2)

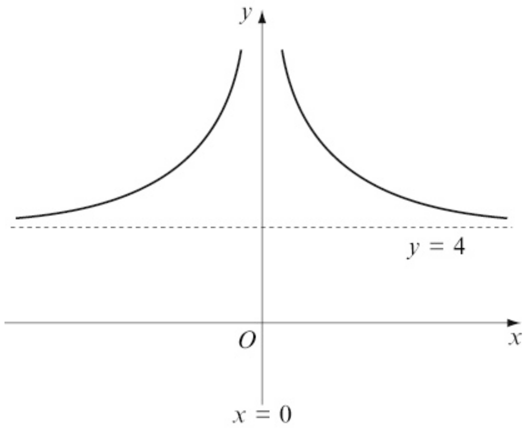
3 b $y = \mathrm{f}(x+2)$

Horizontal translation of -2.



$y = \mathrm{f}(x+2)+4$

Vertical translation of $+4$.

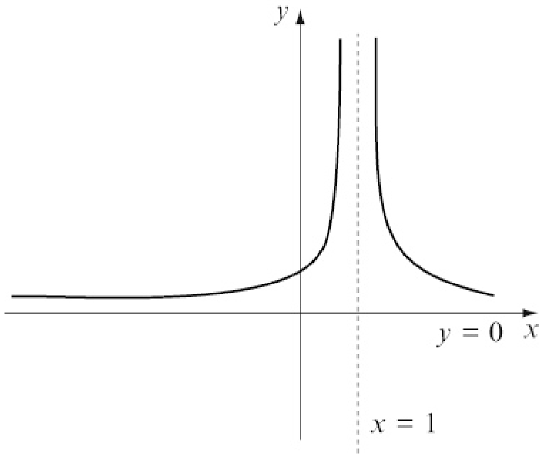


Asymptotes: $x = 0$, $y = 4$

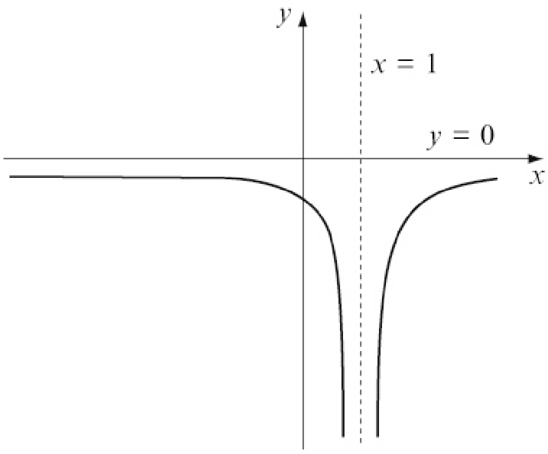
A: (–2, 5)

c $y = \mathrm{f}(2x)$

Horizontal stretch, scale factor $\frac{1}{2}$



$y = -\mathrm{f}(2x)$. Reflection in the x-axis.



Asymptotes: $x = 1$, $y = 0$

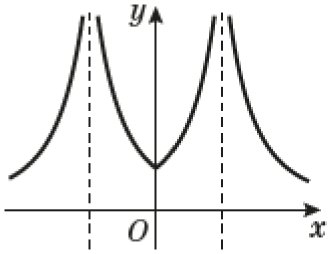
A: (0, –1)

3 d $y = \mathrm{f}(|x|)$ can be written

$$y = \begin{cases} \mathrm{f}(x), & x \geq 0 \\ \mathrm{f}(-x), & x < 0 \end{cases}$$

$y = \mathrm{f}(-x)$ is a reflection of $y = \mathrm{f}(x)$ in the y-axis.

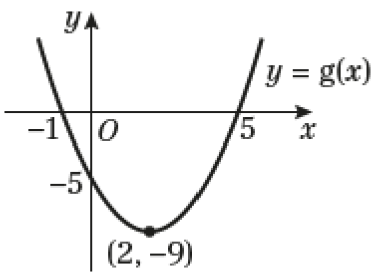
Hence, $y = \mathrm{f}(|x|)$ is the following:



Asymptotes are $x = -2$, $x = 2$ and $y = 0$.

A: (0, 1)

4 a



b **i** (2 + 4, −9 × 2) = (6, −18)

ii $(2 \times \frac{1}{2}, -9) = (1, -9)$

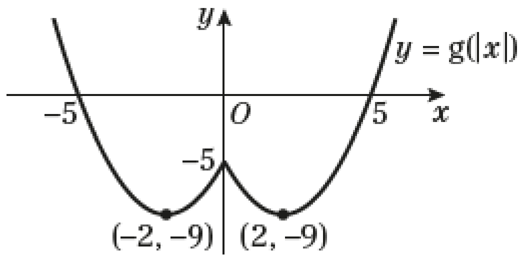
iii (2, −9 × −1) = (2, 9)

c $y = \mathrm{g}(|x|)$ can be written

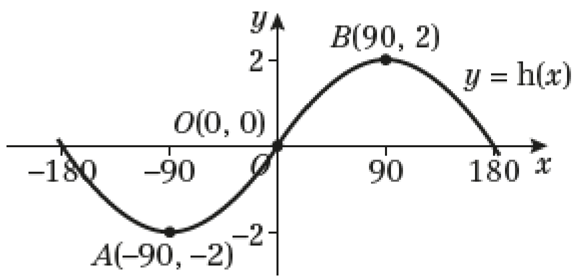
$$y = \begin{cases} \mathrm{g}(x) = (x-2)^2 - 9, & x \geq 0 \\ \mathrm{g}(-x) = (x+2)^2 - 9, & x < 0 \end{cases}$$

$y = \mathrm{g}(-x)$ is a reflection of $y = \mathrm{g}(x)$ in the y-axis.

Hence, $y = \mathrm{g}(|x|)$ is the following:



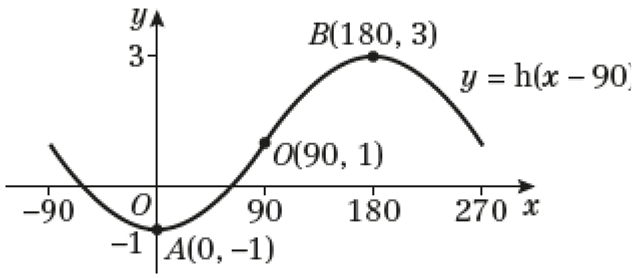
5 a $y = 2\sin x$ is a vertical stretch of $y = \sin x$ by a scale factor 2.



b minimum $A(-90^\circ, -2)$ and maximum $B(90^\circ, 2)$

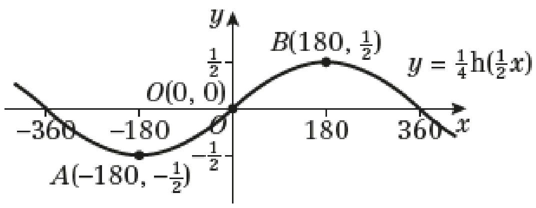
5 c i $h(x-90)$ is a horizontal translation of +90°

$h(x-90)+1$ is a vertical translation of $+1$.



ii $h\left(\frac{1}{2}x\right)$ is a horizontal stretch scale factor 2

$\frac{1}{4}h\left(\frac{1}{2}x\right)$ is a vertical stretch scale factor $\frac{1}{4}$

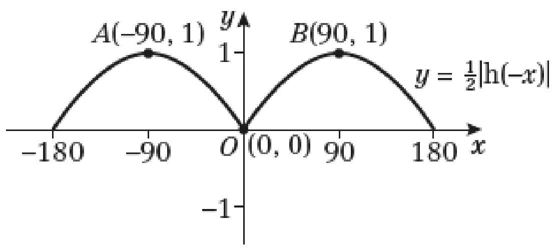


iii $h(-x)$ is a reflection in the y-axis

$|h(-x)|$ causes the part of the graph below the x-axis to be reflected in the x-axis.

$\frac{1}{2}|h(-x)|$ is a vertical stretch scale factor

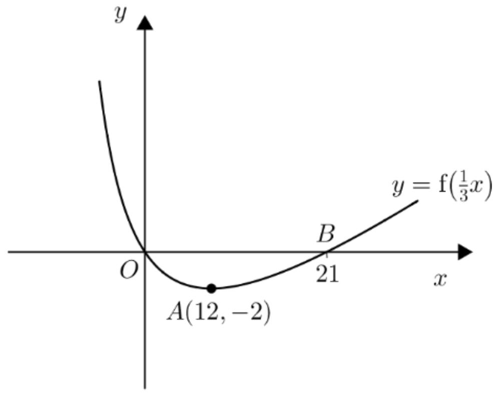
$\frac{1}{2}$



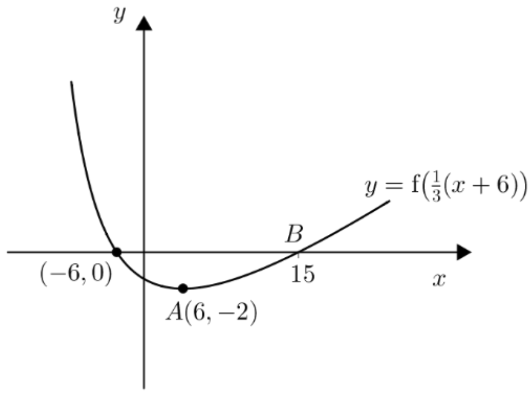
6 $y=f\left(\frac{1}{3}x+2\right)$ can be written as

$y=f\left(\frac{1}{3}(x+6)\right)$

$y=f\left(\frac{1}{3}x\right)$: Horizontal stretch, scale factor 3



$y=f\left(\frac{1}{3}(x+6)\right)$: Horizontal translation of -6



So O is transformed to $(-6,0)$

A is transformed to $(6,-2)$

B is transformed to $(15,0)$