Radians 5E

1 a $\cos\theta = 0.7,\ 0 \leqslant \theta \leqslant 2\pi$

$\cos^{-1} 0.7 = 0.795$

$2\pi - 0.795 = 5.49$

$\theta = 0.795, 5.49$

b $\sin\theta = -0.2,\ 0 \leqslant \theta \leqslant 2\pi$

$\sin^{-1}(-0.2) = -0.201$

$\pi + 0.201 = 3.34$

$2\pi - 0.201 = 6.08$

$\theta = 3.34, 6.08$

c $\tan\theta = 5,\ 0 \leqslant \theta \leqslant 2\pi$

$\tan^{-1} 5 = 1.37$

$\pi + 1.37 = 4.51$

$\theta = 1.37, 4.51$

d $\cos\theta = -1,\ 0 \leqslant \theta \leqslant 2\pi$

$\cos^{-1}(-1) = \pi$

$\theta = \pi$

2 a $4\sin\theta = 3,\ 0 \leqslant \theta \leqslant 2\pi$

$\sin\theta = \frac{3}{4}$

$\sin^{-1}\frac{3}{4} = 0.848$

$\pi - 0.848 = 2.29$

$\theta = 0.848, 2.29$

b $7\tan\theta = 1,\ 0 \leqslant \theta \leqslant 2\pi$

$\tan\theta = \frac{1}{7}$

$\tan^{-1}\frac{1}{7} = 0.142$

$\pi + 0.142 = 3.28$

$\theta = 0.142, 3.28$

c $8\tan\theta = 15,\ 0 \leqslant \theta \leqslant 2\pi$

$\tan\theta = \frac{15}{8}$

$\tan^{-1}\frac{15}{8} = 1.08$

$\pi + 1.08 = 4.22$

$\theta = 1.08, 4.22$

d $\sqrt{5}\cos\theta = \sqrt{2},\ 0 \leqslant \theta \leqslant 2\pi$

$\cos\theta = \frac{\sqrt{2}}{\sqrt{5}}$

$\cos^{-1}\frac{\sqrt{2}}{\sqrt{5}} = 0.886$

$2\pi - 0.886 = 5.40$

$\theta = 0.886, 5.40$

3 a $5\cos\theta + 1 = 3,\ 0 \leqslant \theta \leqslant 2\pi$

$\cos\theta = \frac{2}{5}$

$\cos^{-1}\frac{2}{5} = 1.16$

$2\pi - 1.16 = 5.12$

$\theta = 1.16, 5.12$

b $\sqrt{5}\sin\theta + 2 = 1,\ 0 \leqslant \theta \leqslant 2\pi$

$\sin\theta = -\frac{1}{\sqrt{5}}$

$\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right) = -0.464$

$\pi + 0.464 = 3.61$

$2\pi - 0.464 = 5.82$

$\theta = 3.61, 5.82$

3 c $8\tan\theta - 5 = 5,\ 0 \leqslant \theta \leqslant 2\pi$

$\tan\theta = \frac{10}{8}$

$\tan^{-1}\frac{10}{8} = 0.896$

$\pi + 0.896 = 4.04$

$\theta = 0.896, 4.04$

d $\sqrt{7}\cos\theta - 1 = \sqrt{2},\ 0 \leqslant \theta \leqslant 2\pi$

$\cos\theta = \frac{\sqrt{2}+1}{\sqrt{7}}$

$\cos^{-1}\frac{\sqrt{2}+1}{\sqrt{7}} = 0.421$

$2\pi - 0.421 = 5.86$

$\theta = 0.421, 5.86$

4 a $\sqrt{3}\tan\theta - 1 = 0,\ -\pi \leqslant \theta \leqslant \pi$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\frac{\pi}{6} - \pi = -\frac{5\pi}{6}$

$\theta = -\frac{5\pi}{6}, \frac{\pi}{6}$

b $5\sin\theta = 1,\ -\pi \leqslant \theta \leqslant 2\pi$

$\sin\theta = \frac{1}{5}$

$\sin^{-1}\frac{1}{5} = 0.201$

$\theta = 0.201, 2.94$

c $8\cos\theta = 5,\ -2\pi \leqslant \theta \leqslant 2\pi$

$\cos\theta = \frac{5}{8}$

$\cos^{-1}\frac{5}{8} = 0.896$

$\theta = -0.896, -5.39, 0.896, 5.39$

d $3\cos\theta - 1 = 0.02 - \pi \leqslant \theta \leqslant 3\pi$

$\cos\theta = \frac{1.02}{3} = 0.34$

$\cos^{-1} 0.34 = 1.22$

$\theta = -1.22, 1.22, 5.06, 7.51$

e $0.4\tan\theta - 5 = -7,\ 0 \leqslant \theta \leqslant 4\pi$

$\tan\theta = -\frac{2}{0.4} = -5$

$\tan^{-1}(-5) = -1.37$

$\theta = 1.77, 4.91, 8.05, 11.2$

f $\cos\theta - 1 = -0.82,\ \frac{\pi}{2} \leqslant \theta \leqslant \frac{7\pi}{3}$

$\cos\theta = 0.18$

$\cos^{-1} 0.18 = 1.39$ (not in given interval)

$\theta = 4.89$

5 a $5\cos 2\theta = 4,\ 0 \leqslant \theta \leqslant 2\pi$

Let $X = 2\theta$

$5\cos X = 4,\ 0 \leqslant X \leqslant 4\pi$

$\cos X = \frac{4}{5}$

$X = 0.64, 5.64, 6.92, 11.92$

$\theta = 0.322, 2.82, 3.46, 5.96$

b $5\sin 3\theta + 3 = 1,\ 0 \leqslant \theta \leqslant 2\pi$

Let $X = 3\theta$

$5\sin X + 3 = 1,\ 0 \leqslant X \leqslant 6\pi$

$\sin X = -\frac{2}{5}$

$X = (-0.412), 3.55, 5.87, 9.83, 12.2,$ 16.1, 18.4

$\theta = 1.18, 1.96, 3.28, 4.05, 5.37, 6.15$

5 c $\sqrt{3}\tan 4\theta - 5 = -4,\ 0 \leqslant \theta \leqslant 2\pi$

Let $X = 4\theta$

$\sqrt{3}\tan X - 5 = -4,\ 0 \leqslant X \leqslant 8\pi$

$\tan X = \frac{1}{\sqrt{3}}$

$X = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}, \frac{37\pi}{6}, \frac{43\pi}{6}$

$\theta = \frac{\pi}{24}, \frac{7\pi}{24}, \frac{13\pi}{24}, \frac{19\pi}{24}, \frac{25\pi}{24}, \frac{31\pi}{24}, \frac{37\pi}{24}, \frac{43\pi}{24}$

d $\sqrt{10}\cos 2\theta + \sqrt{2} = 3\sqrt{2},\ 0 \leqslant \theta \leqslant 2\pi$

Let $X = 2\theta$

$\sqrt{10}\cos X + \sqrt{2} = 3\sqrt{2},\ 0 \leqslant X \leqslant 4\pi$

$\cos X = \frac{2\sqrt{2}}{\sqrt{10}} = \frac{2\sqrt{5}}{5}$

$X = 0.464, 5.82, 6.75, 12.1$

$\theta = 0.232, 2.91, 3.37, 6.05$

6 a $\sqrt{2}\sin 3\theta - 1 = 0,\ -\pi \leqslant \theta \leqslant \pi$

Let $X = 3\theta$

$\sqrt{2}\sin X - 1 = 0,\ -3\pi \leqslant X \leqslant 3\pi$

$\sin X = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$X = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

$\theta = -\frac{7\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}$

b $2\cos 4\theta = -1,\ -\pi \leqslant \theta \leqslant 2\pi$

Let $X = 4\theta$

$2\cos X = -1,\ -4\pi \leqslant X \leqslant 8\pi$

$\cos X = -\frac{1}{2}$

$X = -\frac{10\pi}{3}, -\frac{8\pi}{3}, -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}$

$\theta = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$

c $8\tan 2\theta = 7,\ -2\pi \leqslant \theta \leqslant 2\pi$

Let $X = 2\theta$

$8\tan X = 7,\ -4\pi \leqslant X \leqslant 4\pi$

$\tan X = \frac{7}{8}$

$X = -11.8, -8.71, -5.56, -2.42, 0.719, 3.86, 7.00, 10.1$

$\theta = -5.92, -4.35, -2.78, -1.21, 0.359, 1.93, 3.50, 5.07$

d $6\cos 2\theta - 1 = 0.2,\ -\pi \leqslant \theta \leqslant 3\pi$

Let $X = 2\theta$

$6\cos X - 1 = 0.2,\ -2\pi \leqslant X \leqslant 6\pi$

$\cos X = \frac{1.2}{6} = 0.2$

$X = -4.91, -1.37, 1.37, 4.91, 7.65, 11.2, 13.9, 17.5$

$\theta = -2.46, -0.685, 0.685, 2.46, 3.83, 5.60, 6.97, 8.74$

7 a $4\cos^2\theta = 2,\ 0 \leqslant \theta \leqslant 2\pi$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

b $3\tan^2\theta + \tan\theta = 0,\ 0 \leqslant \theta \leqslant 2\pi$

$\tan\theta(3\tan\theta + 1) = 0$

$\tan\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi$

or $\tan\theta = -\frac{1}{3} \Rightarrow \theta = 2.82, 5.96$

c $\cos^2\theta - 2\cos\theta = 3,\ 0 \leqslant \theta \leqslant 2\pi$

$\cos^2\theta - 2\cos\theta - 3 = 0$

$(\cos\theta - 3)(\cos\theta + 1) = 0$

$\cos\theta = 3$ (no solutions)

or $\cos\theta = -1 \Rightarrow \theta = \pi$

d $2\sin^2 2\theta - 5\cos 2\theta = -2,\ 0 \leqslant \theta \leqslant 2\pi$

$2\sin^2 2\theta - 5\cos 2\theta + 2 = 0$

Let $X = 2\theta$

$2\sin^2 X - 5\cos X + 2 = 0,\ 0 \leqslant X \leqslant 4\pi$

$2(1 - \cos^2 X) - 5\cos X + 2 = 0$

$2 - 2\cos^2 X - 5\cos X + 2 = 0$

$2\cos^2 X + 5\cos X - 4 = 0$

$X = \frac{-5 \pm \sqrt{57}}{4}$

$\cos X = -3.14$ (no solutions)

or $\cos X = 0.637 \Rightarrow$

$X = 0.880, 5.40, 7.16, 11.7$

$\theta = 0.440, 2.70, 3.58, 5.84$

8 a $\cos\theta + 2\sin^2\theta + 1 = 0,\ 0 \leqslant \theta \leqslant 2\pi$

$\cos\theta + 2(1 - \cos^2\theta) + 1 = 0$

$\cos\theta + 2 - 2\cos^2\theta + 1 = 0$

$2\cos^2\theta - \cos\theta - 3 = 0$

$(2\cos\theta - 3)(\cos\theta + 1) = 0$

$\cos\theta = 1.5$ (no solutions)

or $\cos\theta = -1 \Rightarrow \theta = \pi$

b $10\sin^2\theta = 3\cos^2\theta,\ 0 \leqslant \theta \leqslant 2\pi$

$\frac{\sin^2\theta}{\cos^2\theta} = \frac{3}{10}$

$\tan^2\theta = 0.3$

$\tan\theta = \pm\sqrt{0.3}$

$\theta = 0.501, 2.64, 3.64, 5.78$

c $4\cos^2\theta + 8\sin^2\theta = 2\sin^2\theta - 2\cos^2\theta,\ 0 \leqslant \theta \leqslant 2\pi$

$6\sin^2\theta = -6\cos^2\theta$

$\tan^2\theta = -1 \Rightarrow$ no solutions

d $2\sin^2\theta - 7 + 12\cos\theta = 0,\ 0 \leqslant \theta \leqslant 2\pi$

$2(1 - \cos^2\theta) - 7 + 12\cos\theta = 0$

$2 - 2\cos^2\theta - 7 + 12\cos\theta = 0$

$2\cos^2\theta - 12\cos\theta + 5 = 0$

$\cos\theta = \frac{12 \pm \sqrt{104}}{4}$

$\cos\theta = 5.55$ (no solutions)

or $\cos\theta = 0.45 \Rightarrow \theta = 1.10, 5.18$

9 a $\cos\left(x-\frac{\pi}{12}\right)=\frac{1}{\sqrt{2}},\ 0\leqslant x<2\pi$

Let $X=x-\frac{\pi}{12}$

$\cos X=\frac{1}{\sqrt{2}},\ -\frac{\pi}{12}\leqslant X<\frac{23\pi}{12}$

$X=\frac{\pi}{4},\frac{7\pi}{4}$

$x=\frac{\pi}{3},\frac{11\pi}{6}$

b $\sin 3x=-\frac{1}{2},\ 0\leqslant x<2\pi$

Let $X=3x$

$\sin X=-\frac{1}{2},\ 0\leqslant X<6\pi$

$X=\left(-\frac{\pi}{6}\right),\frac{7\pi}{6},\frac{11\pi}{6},\frac{19\pi}{6},\frac{23\pi}{6},$

$\frac{31\pi}{6},\frac{35\pi}{6}$

$x=\frac{7\pi}{18},\frac{11\pi}{18},\frac{19\pi}{18},\frac{23\pi}{18},\frac{31\pi}{18},\frac{35\pi}{18}$

10 a $(1+\tan\theta)(5\sin\theta-2),\ -\pi\leqslant\theta<\pi$

$\tan\theta=-1\Rightarrow\theta=-\frac{\pi}{4},\frac{3\pi}{4}$

or $\sin\theta=\frac{2}{5}\Rightarrow\theta=0.412,2.73$

b $4\tan x=5\sin x,\ 0\leqslant x<2\pi$

$4\frac{\sin x}{\cos x}=5\sin x$

$4\sin x=5\sin x\cos x$

$\sin x=0\Rightarrow x=0,\ \pi$

or $\cos x=\frac{4}{5}\Rightarrow x=0.644,\ 5.64$

11 $8\cos^2 x+6\sin x-6=3,\ 0\leqslant x\leqslant 2\pi$

$8(1-\sin^2 x)+6\sin x-6-3=0$

$8-8\sin^2 x+6\sin x-9=0$

$8\sin^2 x-6\sin x+1=0$

$(4\sin x-1)(2\sin x-1)=0$

$\sin x=0.25\Rightarrow x=0.3,2.9$

or $\sin x=0.5\Rightarrow x=0.5,2.6$

12 $\cos^2 x-1=\frac{7}{2}\sin^2 x-2,\ 0\leqslant x\leqslant 2\pi$

$(1-\sin^2 x)-1=\frac{7}{2}\sin^2 x-2$

$\frac{9}{2}\sin^2 x-2=0$

$\sin^2 x=\frac{4}{9}$

$\sin x=\frac{2}{3}\Rightarrow x=0.7,2.4$

or $\sin x=-\frac{2}{3}\Rightarrow x=3.9,5.6$

13 $8\sin^2 x+4\sin x-20=4$

$8\sin^2 x+4\sin x-24=0$

$2\sin^2 x+\sin x-6=0$

Let $Y=\sin x$

$2Y^2+Y-6=0$

$(2Y-3)(Y+2)=0$

$Y=1.5\Rightarrow\sin x=1.5$ (no solutions)

or $Y=-2\Rightarrow\sin x=-2$ (no solutions)

14 a $\tan^2 x - 2\tan x - 6 = 0$

Using the quadratic formula with $a = 1, b = -2$ and $c = -6$ (or by completing the square):

$$\tan x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{2}$$

$$= \frac{2 \pm \sqrt{28}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}}{2}$$

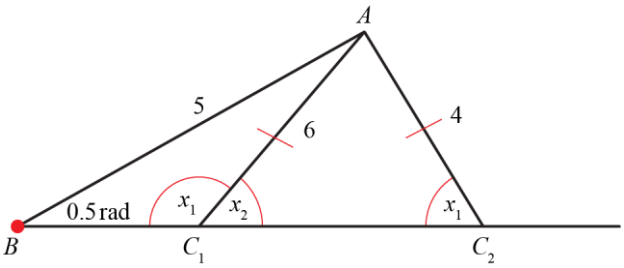
$$= 1 \pm \sqrt{7}$$

So $p = 1$ and $q = 7$

b $\tan\theta = 1 + \sqrt{7} \Rightarrow \theta = 1.3, 4.4, 7.6$

$\tan\theta = 1 - \sqrt{7} \Rightarrow \theta = 2.1, 5.3, 8.4$

15 a



$$\frac{\sin x}{5} = \frac{\sin 0.5}{4}$$

$$\sin x = 5 \times \frac{\sin 0.5}{4}$$

$$= 0.599 \text{ (3 d.p.)}$$

b $x_1 = 0.643\ldots \approx 0.64$ (2 d.p.)

$x_2 = \pi - 0.643 = 2.498\ldots \approx 2.50$ (2 d.p.)