Radians 5F

1 a $\dfrac{\sin 4\theta - \tan 2\theta}{3\theta} \approx \dfrac{4\theta - 2\theta}{3\theta}$

$= \dfrac{2}{3}$

b $\dfrac{1-\cos 2\theta}{\tan 2\theta \sin\theta} \approx \dfrac{1-\left(1-\dfrac{(2\theta)^2}{2}\right)}{2\theta^2}$

$= \dfrac{\dfrac{4\theta^2}{2}}{2\theta^2}$

$= \dfrac{2\theta^2}{2\theta^2}$

$= 1$

c $\dfrac{3\tan\theta - \theta}{\sin 2\theta} \approx \dfrac{3\theta - \theta}{2\theta}$

$= \dfrac{2\theta}{2\theta}$

$= 1$

2 a $\dfrac{\sin 3\theta}{\theta \sin 4\theta} \approx \dfrac{3\theta}{4\theta^2}$

$= \dfrac{3}{4\theta}$

b $\dfrac{\cos\theta - 1}{\tan 2\theta} \approx \dfrac{\left(1-\dfrac{\theta^2}{2}\right)-1}{2\theta}$

$= \dfrac{-\dfrac{\theta^2}{2}}{2\theta}$

$= -\dfrac{\theta}{4}$

c $\dfrac{\tan 4\theta + \theta^2}{3\theta - \sin 2\theta} \approx \dfrac{4\theta + \theta^2}{3\theta - 2\theta}$

$= \dfrac{4\theta + \theta^2}{\theta}$

$= 4 + \theta$

3 a $\cos 0.244 = 0.970379$ (6 d.p.)

b $\cos 0.244 \approx 1 - \dfrac{0.244^2}{2}$

$= 0.970232$

c $\dfrac{0.970232 - 0.970379}{0.970379} \times 100 = -0.015\%$

d $\cos 0.75 = 0.731689$ (6 d.p.)

$\cos 0.75 \approx 1 - \dfrac{0.75^2}{2} = 0.71875$

$\dfrac{0.71875 - 0.731689}{0.731689} \times 100 = -1.77\%$

e The larger the value of θ, the less accurate the approximation is.

4 $\dfrac{\theta - \sin\theta}{\sin\theta} \times 100 = 1$

$(\theta - \sin\theta) \times 100 = \sin\theta$

$100\theta - 100\sin\theta = \sin\theta$

$100\theta = 101\sin\theta$

5 a $\dfrac{4\cos 3\theta - 2 + 5\sin\theta}{1-\sin 2\theta}$

$$\approx \frac{4\left(1-\frac{(3\theta)^2}{2}\right)-2+5\theta}{1-2\theta}$$

$$= \frac{4\left(1-\frac{9\theta^2}{2}\right)-2+5\theta}{1-2\theta}$$

$$= \frac{4-18\theta^2-2+5\theta}{1-2\theta}$$

$$= \frac{2+5\theta-18\theta^2}{1-2\theta}$$

$$= \frac{(1-2\theta)(2+9\theta)}{1-2\theta}$$

$$= 9\theta + 2$$

b When θ is small, 9θ is also small, so

$$\frac{4\cos 3\theta - 2 + 5\sin\theta}{1-\sin 2\theta} \approx 2$$

Challenge

1 a $CD = r\theta = AC \times \theta$

b In the right-anged triangle ABC:

$$\sin\theta = \frac{BC}{AB} \approx \frac{CD}{AC} = \frac{AC\times\theta}{AC} = \theta$$

$$\tan\theta = \frac{BC}{AC} \approx \frac{CD}{AC} = \frac{AC\times\theta}{AC} = \theta$$

2 a For $|x| < 1$,

$$\sqrt{1-x^2}$$

$$= (1-x^2)^{\frac{1}{2}}$$

$$= 1 + \frac{1}{2}(-x^2) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2}(-x^2)^2 + \ldots$$

$$= 1 - \frac{x^2}{2} - \frac{x^4}{8} + \ldots$$

$$\approx 1 - \frac{x^2}{2}$$

b $\cos\theta = \sqrt{1-\sin^2\theta}$

$$\approx 1 - \frac{\sin^2\theta}{2} \text{ since } |\sin\theta| < 1$$

$$\approx 1 - \frac{\theta^2}{2} \text{ since } \sin\theta \approx \theta$$