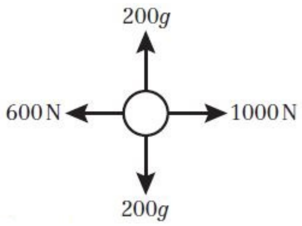
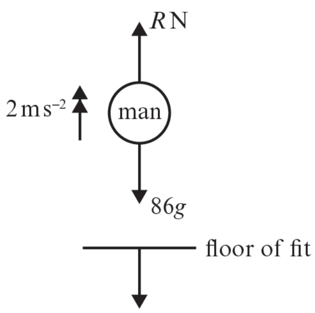
Forces and motion, Mixed exercise 10

1 a



b Vertical forces can be ignored as they are in equilibrium and at right angles to the direction of interest.
$F = ma$
$m = 200$, Resultant force, $F = 1000 - 200 - 400 = 400$
$400 = 200a$
The acceleration of the motorcycle is 2 m s^{-2}.

2



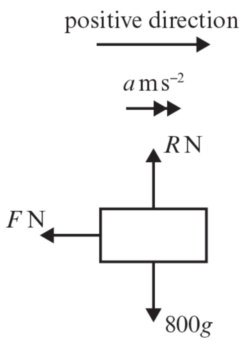
For the man

$$R(\uparrow), \quad R - 86g = 86 \times 2$$
$$R = 86 \times 9.8 + 86 \times 2$$
$$= 1014.8 \approx 1000$$

The reaction on the man on the floor is of equal magnitude to the action of the floor on the man and in the opposite direction.

The force that the man exerts on the floor of the lift is of magnitude 1000 N (2 s.f.) and acts vertically downwards.

3



3 a $u = 18,\ v = 12,\ t = 2.4,\ a = ?$

$$v = u + at$$
$$12 = 18 + 2.4a$$
$$a = \frac{12 - 18}{2.4} = -2.5$$

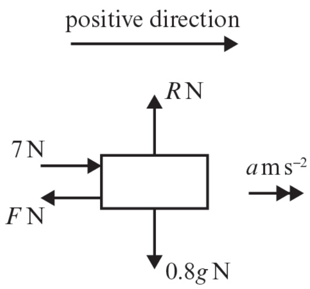
$$F = ma$$
$$-F = 800 \times -2.5 = -2000$$
$$F = 2000\text{ N}$$

b $u = 18,\ v = 12,\ t = 2.4,\ s = ?$

$$s = \left(\frac{u + v}{2}\right)t$$
$$= \left(\frac{18 + 12}{2}\right) \times 2.4$$
$$= 15 \times 2.4 = 36$$

The distance moved by the car is 36 m

4



a $u = 2,\ v = 4,\ s = 4.8,\ a = ?$

$$v^2 = u^2 + 2as$$
$$4^2 = 2^2 + 9.6a$$
$$a = \frac{16 - 4}{9.6} = 1.25$$

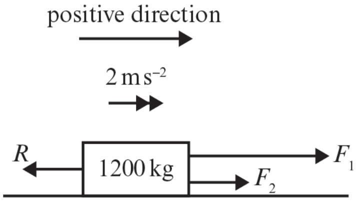
The magnitude of the acceleration of the block is 1.25 m s^{-2}

b $R(\uparrow),\quad F = ma = 0.8 \times 1.25 = 1$

$R(\rightarrow),\quad 7 - F = 6$

The magnitude of the frictional force between the block and the floor is 6 N.

5



Let R = the resistive force
Let F_1 = the driving force
Let F_2 = the resultant force

$F_2 = ma = 1200 \times 2 = 2400$

$F_1 = 3R \Rightarrow R = \frac{1}{3}F_1$

The driving force is the resultant force plus the resistive force:

$F_1 = R + F_2 = \frac{1}{3}F_1 + 2400$

$\frac{2}{3}F_1 = 2400$

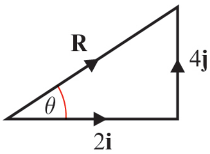
$F_1 = 3600$

The magnitude of the driving force is 3600 N, as required.

6 $\mathbf{F}_1 = (3\mathbf{i} + 2\mathbf{j})$, $\mathbf{F}_2 = (4\mathbf{i} - \mathbf{j})$, $m = 0.25$
$F = \mathbf{F}_1 + \mathbf{F}_2 = ma$
$(3\mathbf{i} + 2\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) = 0.25a$
$(7\mathbf{i} + \mathbf{j}) = 0.25a$
$a = \dfrac{(7\mathbf{i}+\mathbf{j})}{0.25}$
The acceleration is $(28\mathbf{i} + 4\mathbf{j})$ m s^{-2}.

7 $\mathbf{F}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\mathbf{F}_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{F}_3 = \begin{pmatrix} a \\ -2b \end{pmatrix}$ $m = 2,\ a = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
$F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = ma$
$\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} a \\ -2b \end{pmatrix} = 2\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$
Considering **i** components: $2 + 3 + a = 6$
$a = 6 - 5$
Considering **j** components: $-1 - 1 - 2b = 4$
$-2b = 4 + 2$
The values of a and b are 1 and −3, respectively.

8



a $|R| = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
Using $F = ma$
$2\sqrt{5} = 2a$
The acceleration of the sled is $\sqrt{5}$ m s^{-2}.

8 b $u = 0$, $t = 3$, $a = \sqrt{5}$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$s = (0 \times 3) + \left(\frac{1}{2} \times \sqrt{5} \times 3^2\right) = \frac{9\sqrt{5}}{2}$

The sled travels a distance of $\frac{9\sqrt{5}}{2}$ m.

9 a Since object is in equilibrium, $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

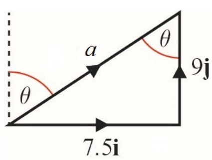
$(3a\mathbf{i} + 4b\mathbf{j}) + (5b\mathbf{i} + 2a\mathbf{j}) + (-15\mathbf{i} - 18\mathbf{j}) = 0$
Collecting **i** terms: $3a + 5b = 15$ **(1)**
Collecting **j** terms: $2a + 4b = 18$ **(2)**
Subtracting (**2**) from (**1**) gives $a + b = -3$
Therefore $b = -3 - a$

Substituting this into (**1**):
$3a + 5(-3 - a) = 15$
$3a - 15 - 5a = 15$
$-2a = 30$
$a = -15$
Substituting this into (**1**):
$3(-15) + 5b = 15$
$5b = 15 + 45 = 60$
$b = 12$

The values of a and b are -15 and 12, respectively.

b i $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$, so when $\mathbf{F}_3$ is removed, the resultant force $F = -\mathbf{F}_3$
i.e. $F = (15\mathbf{i} + 18\mathbf{j})$

$m = 2$
$F = ma$
$(15\mathbf{i} + 18\mathbf{j}) = 2a$
$a = (7.5\mathbf{i} + 9\mathbf{j})$



$|a| = \sqrt{7.5^2 + 9^2} = \sqrt{137.25}$
Using Z angles (see diagram), bearing = θ
$\tan\theta = \frac{7.5}{9}$
The magnitude of the acceleration is 11.7 m s^{-2} and it has a bearing of 039.8° (both to 3 s.f.).

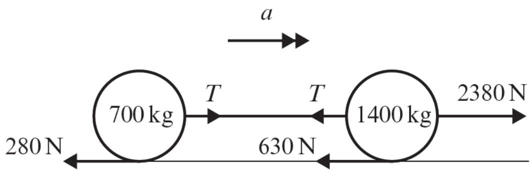
9 b ii $u = 0$, $t = 3$, $a = 11.7$, $s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0 \times 3) + \left(\tfrac{1}{2} \times 11.7 \times 3^2\right) = \frac{105.3}{2}$$

The object travels a distance of 52.7 m (to 3 s.f.).

10



a $F = ma$

For the whole system:

$F = 2380 - 630 - 280 = 1470$

$m = 1400 + 700 = 2100$

$1470 = 2100a$

Since the tow-rope is inextensible, the acceleration of each part of the system is identical.

The acceleration of the car is 0.7 m s^{-2}.

b For the trailer:

$F = T - 280$, $m = 700$, $a = 0.7$

$T - 280 = 700 \times 0.7 = 490$

The tension in the tow-rope is 770 N.

c i For the car, after the rope breaks:

resultant force = 2380 – 630 = 1750

$m = 1400$

therefore $a = 1750 \div 1400 = 1.25$

$u = 12$

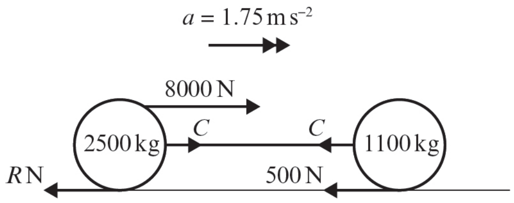
$$s = ut + \tfrac{1}{2}at^2$$

$$s = (12 \times 4) + \left(\tfrac{1}{2} \times 1.25 \times 4^2\right) = 48 + 10$$

In the first 4 s after the tow-rope breaks, the car travels 58 m.

ii Since the tow-rope is inextensible, the tension is constant throughout the length, and the acceleration of each part of the system is identical.

11



a $F = ma$

For the whole system:

$F = 8000 - 500 - R = 7500 - R$

$m = 2500 + 1100 = 3600$

$a = 1.75$

$7500 - R = 3600 \times 1.75 = 6300$

$R = 7500 - 6300$

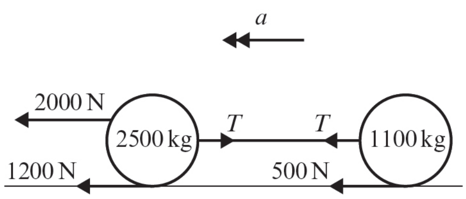
The resistance to the motion of the train is 1200 N, as required.

b Considering the carriage only:

$C - 500 = 1100 \times 1.75 = 1925$

The compression force in the shunt is 2425 N.

c



Taking ← as positive

Deceleration = α

Force on carriage = resistance to motion + thrust in shunt

Using $F = ma$

$500 + C = 1100\alpha$

$$\alpha = \frac{500 + C}{1100}$$

For engine:

$2000 + 1200 - C = 2500\alpha$

Substituting for α:

$$3200 - C = 2500 \times \left(\frac{500 + C}{1100}\right)$$

$$11\cancel{00}(3200 - C) = 25\cancel{00}(500 + C)$$

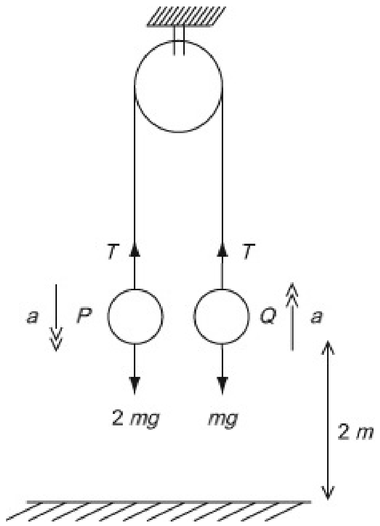
$$35200 - 11C = 12500 + 25C$$

$$35200 - 12500 = 11C + 25C$$

$$C = \frac{22700}{36}$$

The thrust in the shunt is 630 N (2 s.f.).

12 a



For P: $R(\downarrow)$, $2mg - T = 2ma$

For Q: $R(\uparrow)$, $T - mg = ma$

Add, $mg = 3ma$

$a = \frac{1}{3}g \text{ m s}^{-1}$

b For P:

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times \frac{1}{3}g \times 2$

$v = \sqrt{\frac{4g}{3}}$

$= 3.6 \text{ m s}^{-1}$ (2 s.f.)

c For Q:

$R(\uparrow)$, $-mg = ma$

$a = -g$

$v^2 = u^2 + 2as$ $(\uparrow)$,

$0 = \frac{4g}{3} - 2gs$

$s = \frac{2}{3}\text{ m}$

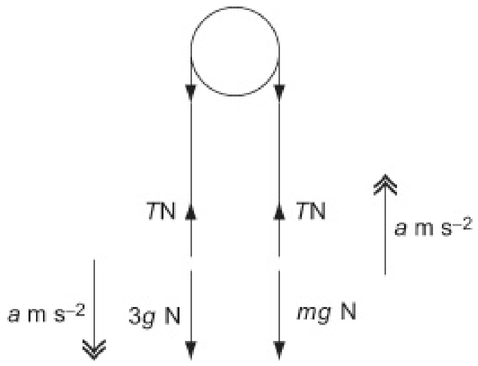
As Q is 4m above the ground when P stops moving $= 4 + s$

$\therefore$ Height above the ground $= 4\frac{2}{3}\text{ m}$

d i In an extensible string $\Rightarrow$ acceleration of both masses is equal.

ii Smooth pulley $\Rightarrow$ same tension in string either side of the pulley.

13 a



For the 3 kg mass

$R(\downarrow), \quad F = ma$

$3g - T = 3 \times \frac{3}{7}g$

$T = 3g - \frac{9}{7}g = \frac{12}{7}g$

The tension in the string is $\frac{12}{7}g$ N

b For the m kg mass

$R(\uparrow), \quad F = ma$

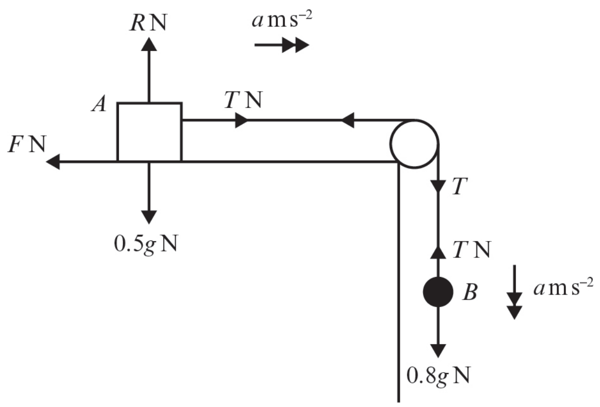
$T - mg = m \times \frac{3}{7}g$

Using the answer to **a**

$\frac{12}{7}g - mg = \frac{3}{7}mg$

$\frac{12}{7} = \frac{10}{7}m \Rightarrow m = 1.2$

14



a For B:

$u = 0, \; s = 0.4, \; t = 0.5, \; a = ?$

$s = ut + \frac{1}{2}at^2$

$0.4 = 0 + \frac{1}{2}a \times 0.5^2 = \frac{1}{8}a$

$a = 8 \times 0.4 = 3.2$

The acceleration of B is $3.2\,\text{m}\,\text{s}^{-2}$

14 b For B:

$$\text{force} = ma$$
$$0.8g - T = 0.8 \times 3.2$$

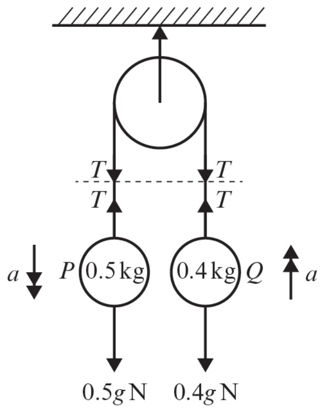
$$T = 0.8 \times 9.8 - 0.8 \times 3.2$$
$$= 5.3$$

The tension in the string is 5.3 N (2 s.f.). (As the numerical value $g = 9.8$ has been used, you should correct your answer to 2 significant figures.)

c $F = 3.7$ (2 s.f.)

d The information that the string is inextensible has been used in part **c** when the acceleration of A has been taken to be equal to the acceleration of B.

15



a **i** For P, $R(\downarrow)$: $0.5g - T = 0.5a$ **(1)**

ii For Q, $R(\uparrow)$: $T - 0.4g = 0.4a$ **(2)**

b (**1**) × 4: $2g - 4T = 2a$

(**2**) × 5: $5T - 2g = 2a$

Equating these:

$2g - 4T = 5T - 2g$

$9T = 4g$

The tension in the string is $\frac{4}{9}g$ N (4.35 N).

c Using equation (**1**):

$\frac{1}{2}g - \frac{4}{9}g = \frac{1}{2}a$

$g - \frac{8}{9}g = a$

The acceleration is $\frac{1}{9}g$ m s^{-2} (1.09 m s^{-2} (3 s.f.)).

15 d When the string breaks, Q has moved up a distance s_1 and reached a speed v_1
Now Q moves under gravity (after the string breaks) initially upwards.
To reach the floor it has to travel a distance $s = 2 + s_1$

While the string is intact, up positive:

$u = 0$, $t = 0.2$, $a = \frac{g}{9}$, $s_1 = ?$

$$s_1 = ut + \frac{1}{2}at^2$$
$$= (0 \times 0.2) + \left(\frac{1}{2} \times \frac{g}{9} \times 0.2^2\right)$$
$$= \frac{g}{450}$$

$$v_1 = u + at$$
$$= 0 + \frac{g}{9} \times 0.2$$
$$= \frac{g}{45}$$

So, when the string breaks, Q is $2 + \frac{g}{450}$ above the ground, a moving upwards with a speed of $\frac{g}{45}$.

After string breaks, Q moves under gravity. So taking down as positive, for the motion after the string breaks, we have

$u = v_1 = -\frac{g}{45}$, $a = g$, $s = 2 + \frac{g}{450}$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$2 + \frac{g}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$
$$\frac{(900+g)}{450} = -\frac{g}{45}t + \frac{1}{2}gt^2$$
$$0 = \frac{1}{2}gt^2 - \frac{g}{45}t - \frac{(900+g)}{450}$$

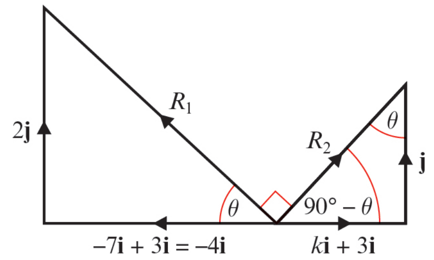
Let $g = 9.8 \Rightarrow 4.9t^2 - 0.217\dot{8}t - 2.02178 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-0.2178 \pm \sqrt{(-0.2178)^2 - (4 \times 4.9 \times -2.02178)}}{2 \times 4.9}$$
$$= \frac{-0.218 \pm \sqrt{39.674}}{9.8}$$
$$= 0.66\text{ s or } -0.621\text{ s}$$

Only the positive root is relevant: $t = 0.66$ (2 s.f.)
Q hits the floor 0.66 s after the string breaks.

Challenge

Total force on first boat: $R_1 = (-7\mathbf{i} + 2\mathbf{j}) + 3\mathbf{i} = -4\mathbf{i} + 2\mathbf{j}$
Total force on second boat: $R_2 = (k\mathbf{i} + \mathbf{j}) + 3\mathbf{i} = (k + 3)\mathbf{i} + \mathbf{j}$
Since mass is a vector quantity, the acceleration of each boat will be parallel to the resultant force acting on it, so the relationship between the components of the accelerations is as shown in the diagram below.



From R_1: $\tan\theta = \frac{2}{4} = \frac{1}{2}$

From R_2: $\tan\theta = \frac{k+3}{1} = k+3$

Equating these: $\frac{1}{2} = k+3$

$2k + 6 = 1$

$2k = -5$

The value of k is -2.5.

