Measures of location and spread 2C

1 a $Q_2 = \frac{16+1}{2}$ th value = 8.5th value

1009, 1013, 1014, 1017, 1017, 1017, 1018, 1019, 1021, 1022, 1024, 1024, 1025, 1027, 1029, 1031

$Q_2 = \frac{1019+1021}{2} = 1020$ hPa

b Q_1 = 4.5th value
Q_1 = 1017 hPa

Q_3 = 12.5th value
Q_3 = 1024.5 hPa

2 $Q_2 = \frac{95+1}{2}$ th value = 48th value
Q_2 = 37

Q_1 = 24th value
Q_1 = 37

Q_3 = 72nd value
Q_3 = 38

3 In this case, the number of breakdowns is a discrete variable which in this situation has been treated as a continuous variable. While not ideal, it is the best you can do.

Median value is the 13th value. This is in the first class.
Let m be the median.

$\frac{m-0}{1.5-0} = \frac{13-0}{18-0}$ so $m \approx 1.08$ (3 sf)

4 a Median: $\frac{31}{2}$ = 15.5th value
Using interpolation:

$\frac{Q_2 - 399.5}{449.5-399.5} = \frac{15.5-9}{19-9}$

Q_2 = 432 kg

4 b $Q_1 : \frac{31}{4} = 7.75$th value, so Q_1 is in class $350 - 399$

$$\frac{Q_1 - 349.5}{399.5 - 349.5} = \frac{7.75 - 3}{9 - 3}$$

$$\frac{Q_1 - 349.5}{50} = \frac{4.75}{6}$$

$$Q_1 = 39.58 + 349.5 = 389$$

c $Q_3 : 3 \times \frac{31}{4} = 23.25$th value, so Q_3 is in class $450 - 499$

$$\frac{Q_3 - 449.5}{499.5 - 449.5} = \frac{23.25 - 19}{26 - 19}$$

$$\frac{Q_3 - 449.5}{50} = \frac{4.25}{7}$$

$$Q_3 = 30.36 + 449.5 = 480$$

d Three-quarters of the cows weigh less than 480 kg.

5 a Estimate for the mean $= \frac{(25 \times 6) + (35 \times 10) + (45 \times 18) + (55 \times 13) + (65 \times 2)}{49}$

$$= \frac{2155}{49}$$

$$= 44.0 \text{ minutes } (\text{to 3 s.f.})$$

b $65\text{th} : \frac{65}{100} \times 49 = 31.85$

$$\frac{P_{65} - 40}{50 - 40} = \frac{31.85 - 16}{34 - 16}$$

$P_{65} = 48.8$

c $$\frac{P_{90} - 50}{60 - 50} = \frac{44.1 - 34}{47 - 34}$$

$P_{90} = 57.8$

90th percentile = 57.8 minutes, so more than 10% of customers have to wait longer then 57.8 minutes – not 56 minutes as stated by the firm.

6 a $\dfrac{P_{80}-2.5}{3.0-2.5}=\dfrac{80-61}{89-61}$

$P_{80} = 2.84$ (to 2 d.p.)

80th percentile = 2.84 m, so 80% of condors have a wingspan of less than 2.84 m.

b The 90th percentile is in the $3.0 \leq w$ class. There is no upper boundary for this class, so it is not possible to estimate the 90th percentile.