Measures of location and spread, Mixed Exercise 2

1 $(8\times 65)+(12\times 72)=1384$

$\frac{1384}{20}=69.2$ marks

2 a 10, 12, 9, 2, 2.5, 9.5

b coded mean $=\frac{45}{6}=7.5$

c mean $= 7.5 \times 80 + 7 = 607$

3 $18\times 1000+720=£18720$

4 a Group *A*:

$(1 \times 24.5) + (3 \times 34.5) + (6 \times 44.5) + (6 \times 54.5) + (11 \times 64.5) + (10 \times 74.5) + (8 \times 84.5)$

Mean $=\frac{2852.5}{45}=63.39$ marks

Group *B*:

$(1 \times 24.5) + (2 \times 34.5) + (4 \times 44.5) + (13 \times 54.5) + (15 \times 64.5) + (6 \times 74.5) + (3 \times 84.5)$

Mean $=\frac{2648}{44}=60.18$ marks

b The method used to teach group *A* is best as the mean mark is higher.

5 a Modal Class is $21-25$ hours

b We need the 40th value. This is in the $21-25$ class.

$\frac{m-20.5}{25.5-20.5}=\frac{40-30}{75-30}$

$45m=50+922.5=972.5$

median = 21.6 hours

c mean $= 20.6$ hours

d $12\times 22.3=267.6$ hours

Total hours for all 92 batteries is $267.6+1645=1912.6$
Mean life for 92 batteries is 20.8 hours.

6 CF = 5 15 30 42 50

6 Data is continuous, so:

$Q_1 : \frac{50}{4} = 12.5$th value, so Q_1 is in class $21 - 40$

$$\frac{Q_1 - 20.5}{40.5 - 20.5} = \frac{12.5 - 5}{15 - 5}$$

$$\frac{Q_1 - 20.5}{20} = \frac{7.5}{10}$$

$$Q_1 = 35.5$$

$Q_3 : \frac{3 \times 50}{4} = 37.5$th value, so Q_3 is in class $61 - 80$

$$\frac{Q_3 - 60.5}{80.5 - 60.5} = \frac{37.5 - 30}{42 - 30}$$

$$Q_3 = 73$$

$$\text{IQR} = 73 - 35.5 = 37.5$$

7 a $30\text{th} : \frac{30}{100} \times 100 = 30$

$$P_{30} = 20.5$$

b $70\text{th} : \frac{70}{100} \times 100 = 70$

$$\frac{P_{70} - 30.5}{40.5 - 30.5} = \frac{70 - 60}{84 - 60}$$

$$P_{66} = 34.7$$

c 30% to 70% interpercentile $= 34.7 - 20.5 = 14.2$

8 a $Q_1 : \frac{80}{4} = 20$th value, so Q_1 is in class $40 - 49$

$$\frac{Q_1 - 39.5}{49.5 - 39.5} = \frac{20 - 15}{51 - 15}$$

$$\frac{Q_1 - 39.5}{10} = \frac{5}{36}$$

$$Q_1 = 40.9$$

8 a Q_3: $\frac{3\times 80}{4} = 60$th value, so Q_3 is in class $50-59$

$$\frac{Q_3 - 49.5}{59.5 - 49.5} = \frac{60-51}{71-51}$$

$$Q_3 = 54$$

$$\text{IQR} = 54 - 40.9 = 13.1$$

b $\text{Variance} = \frac{183040}{80} - \left(\frac{3740}{80}\right)^2 = 102.4375 = 102$

$\text{Standard deviation} = \sqrt{102.4375} = 10.1$

9 CF = 5 15 41 49 50

a Data is continuous, so:

Q_1: $\frac{50}{4} = 12.5$th value, so Q_1 is in class $95-100$

$$\frac{Q_1 - 95}{100-95} = \frac{12.5-5}{15-5}$$

$$Q_1 = 98.75$$

b Q_3: $3\times\frac{50}{4} = 37.5$th value, so Q_3 is in class $100-105$

$$\frac{Q_3 - 100}{105-100} = \frac{37.5-15}{41-15}$$

$$Q_3 = 104.33$$

c $\text{IQR} = 104.33 - 98.75 = 5.58$

d $\text{Standard deviation} = \sqrt{\frac{516112.5}{50} - \left(\frac{5075}{50}\right)^2} = 4.47$

10 a $\text{Mean} = \frac{(12\times 12)+(14\times 14)+(16\times 4)}{30} = \frac{202}{15} = 13.5$ (1 d.p)

$\text{Standard deviation} = \sqrt{\frac{916}{5} - \left(\frac{202}{15}\right)^2} = 1.36$ (3 s.f.)

10 b $10\text{th}: \frac{10}{100} \times 30 = 3$

$$\frac{P_{10} - 11}{13 - 11} = \frac{3 - 0}{12 - 0}$$

$$P_{10} = 11.5$$

$90\text{th}: \frac{90}{100} \times 30 = 27$

$$\frac{P_{90} - 15}{17 - 15} = \frac{27 - 26}{30 - 26}$$

$$P_{90} = 15.5$$

10% to 90% interpercentile range $= 15.5 - 11.5 = 4°C$

c $13.5 + 1.36 = 14.86$

Using interpolation:

$$\frac{14.86 - 13}{15 - 13} = \frac{d - 12}{26 - 12}$$

$d = 25.02$

$30 - d = 4.98$

So 5 days

11 a Coded mean $= \frac{106}{31} = 3.419...$

Coded standard deviation $= \sqrt{\frac{80.55}{31}} = 1.6119...$

b Mean $= 3.419... \times 2 + 3 = 9.84$ kn

Standard deviation $= 1.6119 \times 2 = 3.22$ kn

12 a Mean $= \frac{316}{20} = 15.8$

Standard deviation $= \sqrt{\frac{5078}{20} - \left(\frac{316}{20}\right)^2} = 2.06$

b It will decrease the mean wing span since $13 < 15.8$

c Coded mean $= \frac{104}{20} = 5.2$

Mean $= 5.2 \times 10 + 5 = 57$ cm

Coded standard deviation $= \sqrt{\frac{1.8}{20}} = 0.3$

Standard deviation $= 0.3 \times 10 = 3$

Challenge

Total = $3.1 \times 20 = 62$
New total = $62 - 2.3 + 3.2 = 62.9$
New mean = $\frac{62.9}{20} = 3.145$

$\sigma = 1.4$, $\sigma^2 = 1.96$

$1.96 = \frac{\sum x^2}{20} - \left(\frac{62}{20}\right)^2$
$\sum x^2 = 231.4$
New $\sum x^2 = 231.4 - 2.3^2 + 3.2^2 = 236.35$
New standard deviation = $\sqrt{\frac{236.35}{20} - \left(\frac{62.9}{20}\right)^2} = 1.39$