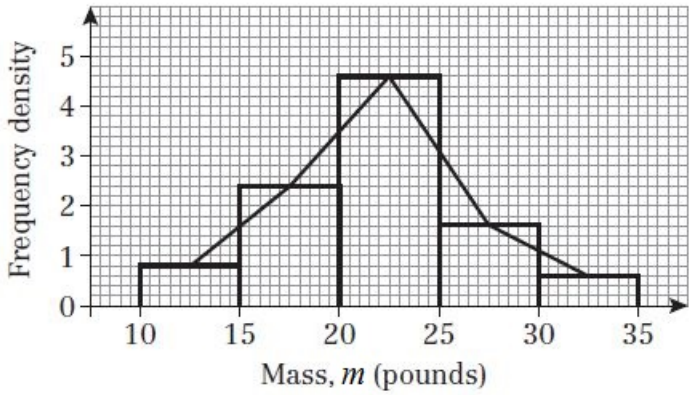
Representations of data 3D

1 a Class widths are all 5.

Frequency densities are: 0.8, 2.4, 4.6, 1.6, 0.6



2 a Time is a continuous variable.

b Frequency = frequency density × class width
= 5 × 20 = 100

So there were 100 students who took between 40 and 60 seconds.

c 100 students took between 40 and 60 seconds (from part **b**)
6 × 10 = 60 students took between 60 and 70 seconds.
8.6 × 10 = 86 students took between 70 and 80 seconds.

So 100 + 60 + 86 = 246 students took 80 seconds or less.

d 246 students took 80 seconds of less (from part **c**)
14 × 5 = 70 students took between 80 and 85 seconds.
12 × 5 = 60 students took between 85 and 90 seconds.
3 × 30 = 90 students took between 90 and 120 seconds.

Total: 246 + 70 + 60 + 90 = 466
There are 466 students in total.

3 a Distance is a continuous variable.

b Frequency = frequency density × class width
0 m to 20 m: 2 × 20 = 40
20 m to 35 m: 5 × 15 = 75
35 m to 45 m: 10 × 10 = 100
45 m to 60 m: 6 × 15 = 90
60 m to 65 m: 1 × 5 = 5

Total: 40 + 75 + 100 + 90 + 5 = 310

So 310 people entered the competition.

3 c Frequency = frequency density × class width
30 m to 35 m: 5 × 5 = 25
35 m to 40 m: 10 × 5 = 50

Total: 25 + 50 = 75

So 75 people threw between 30 and 40 metres.

d From part **b**:
45 m to 60 m: 90 people
60 m to 65 m: 5 people

Total: 90 + 5 = 95

So 95 people threw between 45 and 60 metres.

e From part **b**, 40 people threw between 0 and 20 metres.
20 m to 25 m: 5 × 5 = 25

Total: 40 + 25 = 65

So 65 people threw between 0 and 25 metres.

4 a The bar for $28 \leqslant m < 32$ has an area of 10 × 10 = 100 squares.
If 100 squares represents 32 lambs then
$\frac{100}{4}$ squares represents $\frac{32}{4}$ lambs.
i.e. 25 squares represents 8 lambs.

b The class $24 \leqslant m < 26$ contains 5 × 20 = 100 squares.
As above, this represents 32 lambs.

c The class $20 \leqslant m < 24$ contains 10 × 10 = 100 squares which represents 32 lambs.

The class $24 \leqslant m < 26$ contains 5 × 20 = 100 squares which represents 32 lambs.

The class $26 \leqslant m < 28$ contains 5 × 40 = 200 squares which represents 64 lambs.

The class $28 \leqslant m < 32$ contains 10 × 10 = 100 squares which represents 32 lambs.

The class $32 \leqslant m < 34$ contains 5 × 5 = 25 squares which represents 8 lambs.

So in total we have 32 + 32 + 64 + 32 + 8 = 168 lambs.

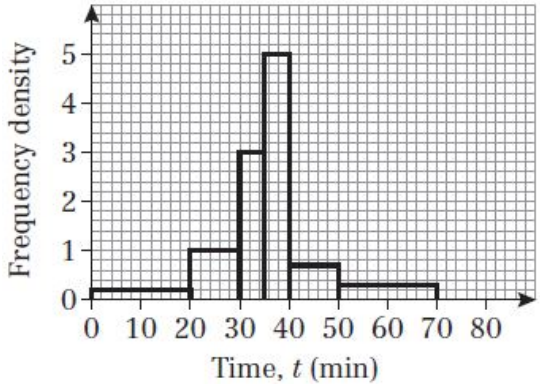
d Class $25 \leqslant m < 26$ is approximately $\frac{1}{2}$ of class $24 \leqslant m < 26$ which equates to 16 lambs.
Class $26 \leqslant m < 28$ represents 64 lambs.
Class $28 \leqslant m < 29$ is approximately $\frac{1}{4}$ of class $28 \leqslant m < 32$ which equates to 8 lambs.

So in total we have 16 + 64 + 8 = 88 lambs.

5 a i Use extra columns to help, using the frequency densities given in the histogram:

Time, t (min)	Frequency	Class width	Frequency density
$0 \leqslant t < 20$	4	20	0.2
$20 \leqslant t < 30$	$10 \times 1 = 10$	10	1
$30 \leqslant t < 35$	15	5	3
$35 \leqslant t < 40$	25	5	5
$40 \leqslant t < 50$	$10 \times 0.7 = 7$	10	0.7
$50 \leqslant t < 70$	$20 \times 0.3 = 6$	20	0.3

5 a ii



b $\left(\frac{5}{10}\times 10\right)+15+\left(\frac{3}{5}\times 25\right)=35$ passengers.

6 a 12.5 and 14.5 are the class boundaries, as we are dealing with continuous data.

b i The class boundaries for the 15–17 class are 14.5 and 17.5.
This width is 1.5 times the width of the 13–14 class, since 17.5 – 14.5 = 3 = 1.5 x 2.
So the width of the class is $1.5 \times 4 = 6$ cm.

ii The frequency density for the 13–14 class is $\frac{24}{2}=12$.

The frequency density of this class is 6, which is 0.5 times the frequency density above: 12.
So the height of the class is 0.5 x 6 = 3 cm.

7 a Width is half of the $8 \leqslant t < 10$ class, which is 0.5 cm.

Height is double the frequency density, so must be $\frac{7}{1} \times 2 = 14$ cm.

b Mean = $\frac{\sum fx}{\sum f} = \frac{645}{62} = 10.4$°C (to 3 s.f.) where x is taken as the midpoint of each class.

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{113.89 - \left(\frac{645}{62}\right)^2} = 2.4$ (to 2 s.f.)

c Q_1 is the $\frac{31}{4} = 7.75^{\text{th}}$ piece of data.

$\frac{Q_1 - 8}{10 - 8} = \frac{7.75 - 4}{12 - 4}$ using linear interpolation on the $8 \leq t < 10$ class.

$Q_1 = 8.94$°C

d Mean + standard deviation = 12.8°C

$\frac{12.8 - 12}{15 - 12} = \frac{d - 25}{30 - 25}$

$d = 26.33$ days
So 4.7 days