Statistical distributions 6A

1 a This is not a discrete random variable, since height is a continuous quantity.

b This is a discrete random variable, since it is takes whole number values at random.

c This is not a discrete random variable, since the number of days in a given week is always 7; the result is predetermined and so not random.

2 {0, 1, 2, 3, 4}

3 a (2, 2), (2, 3), (3, 2), (3, 3)

b i

x	4	5	6
$\mathbf{P}(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

ii

$$\mathrm{P}(X = x) = \begin{cases} \frac{1}{4}, \text{ if } x = 4, 6 \\ \frac{1}{2}, \text{ if } x = 5 \\ 0, \text{ otherwise} \end{cases}$$

4
$$\frac{1}{3}+\frac{1}{3}+k+\frac{1}{4}=1$$
$$k=1-\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{4}\right)$$
$$=1-\frac{11}{12}$$
$$=\frac{1}{12}$$

5

x	1	2	3	4
$\mathbf{P}(X = x)$	k	$2k$	$3k$	$4k$

$$k+2k+3k+4k=1$$
$$10k=1$$
$$k=\tfrac{1}{10}$$

6 a

x	1	2	3	4
$\mathbf{P}(X = x)$	k	k	$3k$	$3k$

6 a Using the fact that the probabilities add up to 1:

$$k + k + 3k + 3k = 1$$
$$8k = 1$$
$$k = \tfrac{1}{8}$$

b The probability distribution is:

x	1	2	3	4
$\mathbf{P}(X = x)$	0.125	0.125	0.375	0.375

$\mathrm{P}(X > 1) = 0.125 + 0.375 + 0.375 = 0.875$

7 a

x	−2	−1	0	1	2
$\mathbf{P}(X = x)$	0.1	0.1	β	β	0.2

The probabilities add up to 1.

$$0.1 + 0.1 + \beta + \beta + 0.2 = 1$$
$$2\beta = 1 - 0.4 = 0.6$$
$$\beta = 0.3$$

b

x	−2	−1	0	1	2
$\mathbf{P}(X = x)$	0.1	0.1	0.3	0.3	0.2

c $\mathrm{P}(-1 \leqslant X < 2) = 0.1 + 0.3 + 0.3 = 0.7$

8 $\frac{1}{4} - a + a + \frac{1}{2} + a = 1$

$\frac{3}{4} + a = 1$

$a = \frac{1}{4}$

9 a $\mathrm{P}(X = 1) = \frac{1}{50}$

since each of the 50 individual outcomes is equally likely.

b $\mathrm{P}(X \geqslant 28) = 1 - \frac{27}{50} = \frac{23}{50}$

9 c $P(13 < X < 42) = P(14 \leqslant X \leqslant 41) = \frac{28}{50} = \frac{14}{25}$

10 a $P(1 < X \leq 3) = P(X = 2) + P(X = 3) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

b $P(X < 2) = P(X = 0) + P(X = 1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

c $P(X > 3) = 0$

11 a

s	1	2	3	4
$P(S = s)$	$\frac{2}{3}$	$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$	$\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

b $P(S > 2) = \frac{2}{27} + \frac{1}{27} = \frac{1}{9}$

12 a

x	0	1	2	3	4	5
$P(X = x)$	$0.6^5 = 0.07776$	$0.4 \times 0.6^4 \times 5 = 0.2592$	$0.4^2 \times 0.6^3 \times 10 = 0.3456$	$0.4^3 \times 0.6^2 \times 10 = 0.2304$	$0.4^4 \times 0.6 \times 5 = 0.0768$	$0.4^5 = 0.01024$

b

y	0	1	2	3	4	5
$P(Y = y)$	$0.8^5 = 0.32768$	$0.2 \times 0.8^4 \times 5 = 0.4096$	$0.2^2 \times 0.8^3 \times 10 = 0.2048$	$0.2^3 \times 0.8^2 \times 10 = 0.0512$	$0.2^4 \times 0.8 \times 5 = 0.0064$	$0.2^5 = 0.00032$

c

z	1	2	3	4	5
$P(Z = z)$	0.4	$0.4 \times 0.6 = 0.24$	$0.4 \times 0.6^2 = 0.144$	$0.4 \times 0.6^3 = 0.0864$	$0.4 \times 0.6^4 + 0.6^5 = 0.1296$

13 a

x	2	3	4
$P(X = x)$	$\frac{1}{2}$	$\frac{2}{9}$	$\frac{1}{8}$

$\frac{1}{2} + \frac{2}{9} + \frac{1}{8} = \frac{61}{72}$

The sum of the probabilities is not 1.

13 b

x	2	3	4
$\mathbf{P}(X = x)$	$\frac{k}{4}$	$\frac{k}{9}$	$\frac{k}{16}$

$$\frac{k}{4} + \frac{k}{9} + \frac{k}{16} = 1$$

$$\frac{61k}{144} = 1$$

$$k = \frac{144}{61} = 2\frac{22}{61}$$

Challenge

x	1	2	3	4	5	6	7	8
$\mathbf{P}(X = x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

y	2	3	6
$\mathbf{P}(Y = y)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$\mathrm{P}(X > Y) = \mathrm{P}(X > 2 \text{ and } Y = 2) + \mathrm{P}(X > 3 \text{ and } Y = 3) + \mathrm{P}(X > 6 \text{ and } Y = 6)$

$$= \frac{6}{8} \times \frac{1}{2} + \frac{5}{8} \times \frac{1}{3} + \frac{2}{8} \times \frac{1}{6} = \frac{5}{8}$$