Hypothesis testing 7D

1 $H_0 : p = 0.5$ $H_1 : p \neq 0.5$

If H_0 is true $X \sim B(30, 0.5)$

Expected value would be $30 \times 0.5 = 15$.
The observed value, 10, is less than this so consider $P(X \leq 10)$

$P(X \leq 10) = 0.0494 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.5$

2 H_0: $p = 0.3$ H_1: $p \neq 0.3$

If H_0 is true $X \sim B(25, 0.3)$

Expected value would be $25 \times 0.3 = 7.5$.
The observed value, 10, is more than this so consider $P(X \geq 10)$

$P(X \geq 10) = 0.1894... > 0.05$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.3$

3 H_0: $p = 0.75$ H_1: $p \neq 0.75$

If H_0 is true $X \sim B(10, 0.75)$

Expected value would be $10 \times 0.75 = 7.5$.
The observed value, 9, is more than this so consider $P(X \geq 9)$

$P(X \geq 9) = 0.2440... > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.75$

4 H_0: $p = 0.6$ H_1: $p \neq 0.6$

If H_0 is true $X \sim B(20, 0.6)$

Expected value would be $20 \times 0.6 = 12$
The observed value, 1, is less than this so consider $P(X \leq 1)$

$P(X \leq 1) = 0.00000034.. < 0.005$ (two-tailed)

Reject H_0. There is evidence that $p \neq 0.6$.

5 H_0: $p = 0.02$ H_1: $p \neq 0.02$

If H_0 is true $X \sim B(50, 0.02)$

Expected value would be $50 \times 0.02 = 1$
The observed value, 4, is more than this so consider $P(X \geq 4)$

$P(X \geq 4) = 0.01775... > 0.01$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to doubt $p = 0.02$

6 The probability that an unbiased coin lands on heads is 0.5
X is the number of times the coin being tested lands on heads
p is the probability that the coin being tested lands on heads.

$H_0: p = 0.5$ $H_1: p \neq 0.5$

If H_0 is true $X \sim B(20, 0.5)$

Expected value would be $20 \times 0.5 = 10$
The observed value, 6, is less than this so consider $P(X \leq 6)$

$P(X \leq 6) = 0.0577 > 0.025$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to think that the coin is biased.

7 a $H_0: p = 0.20$ $H_1: p \neq 0.20$

If H_0 is true $X \sim B(20, 0.20)$

$P(X \leq 1) = 0.0692$
$P(X \leq 0) = 0.0115$ (closer to 0.025)

critical value $= 0$

$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9900 = 0.0100$
$P(X \geq 8) = 1 - P(X \leq 7) = 1 - 0.9679 = 0.0321$ (closer to 0.025)

Critical region $X = 0$ and $X \geq 8$

b Actual significance level is $0.0115 + 0.0321 = 0.0436 = 4.36\%$

c $X = 8$ is in the critical region. There is enough evidence to reject H_0. The hospital's proportion of complications differs from the national figure.

8 **a** The probability that a glass bowl made using the original process is cracked is 0.1
X is the number of bowls in the sample using the new process that are cracked.
p is the probability that a bowl made using the new process is cracked.

$H_0 : p = 0.1 \qquad H_1 : p \neq 0.1$

If H_0 is true $X \sim B(20, 0.1)$

Expected value would be $20 \times 0.1 = 2$
The observed value, 1, is less than this so consider $P(X \leq 1)$

$P(X \leq 2) = 0.3917... > 0.05$ (two-tailed)

There is insufficient evidence to reject H_0 so there is no reason to think that the proportion of cracked bowls has changed.

b Double the calculated probability to find the p-value
p- value = 0.3917…+ 0.3917… = 0.7835

9 The probability that a carrot grown in the original fertiliser is longer than 7 cm is 0.25
X is the number of carrots in the sample grown in the new fertiliser that are longer than 7 cm.
p is the probability that a carrot grown in the new fertiliser is longer than 7 cm.

$H_0 : p = 0.25 \qquad H_1 : p \neq 0.25$

If H_0 is true $X \sim B(30, 0.25)$

Expected value would be $30 \times 0.25 = 7.5$
The observed value, 13, is more than this so consider $P(X \geq 13)$

$P(X \geq 13) = 0.02159... < 0.025$ (two-tailed)

There is evidence to reject H_0. Therefore, there is reason to doubt $p = 0.25$.
So the probability of a carrot being longer than 7 cm has changed.

10 The probability that a standard blood test diagnoses the disease is 0.96
X is the number of patients correctly diagnosed in the sample using the new process.
p is the probability that a patient is correctly diagnosed using the new process.

$H_0 : p = 0.96 \qquad H_1 : p \neq 0.96$

If H_0 is true $X \sim B(75, 0.96)$

Expected value would be $75 \times 0.96 = 72$
The observed value, 63, is less than this so consider $P(X \leq 63)$

$P(X \leq 63) = 0.0000417... < 0.05$ (two-tailed)

There is evidence to reject H_0. Therefore, there is reason to doubt $p = 0.96$.
So the new test does not have the same probability of success as the old test.