Constant acceleration 9E

1 a Take downwards as the positive direction.

$s = 28$, $u = 0$, $a = 9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$28 = 0 \times t + \tfrac{1}{2} \times 9.8 \times t^2 = 4.9t^2$$
$$t = \sqrt{\frac{28}{4.9}} = 2.4 \text{ (to 2 s.f.)}$$

The time taken for the diver to hit the water is 2.4 s.

b $v^2 = u^2 + 2as$
$v^2 = 0 + 2 \times 9.8 \times 28 = 548.8$
$v = \sqrt{548.8} = 32.4$ (to 3 s.f.)

When the diver hits the water, he is travelling at 32.4 m s^{-1}.

2 Take upwards as the positive direction.

$u = 20$, $a = -9.8$, $s = 0$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$0 = 20t - 4.9t^2 = t(20 - 4.9t), \quad t \neq 0$$
$$t = \tfrac{20}{4.9} = 4.1 \text{ (to 2 s.f.)}$$

The time of flight of the particle is 4.1 s.

3 Take downwards as the positive direction.

$u = 18$, $a = 9.8$, $t = 1.6$, $s = ?$

$$s = ut + \tfrac{1}{2}at^2 = 18 \times 1.6 + 4.9 \times 1.6^2 = 41 \text{ (to 2 s.f.)}$$

The height of the tower is 41 m.

4 a Take upwards as the positive direction.

$u = 24$, $a = -9.8$, $v = 0$, $s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 24^2 - 2 \times 9.8 \times s$$
$$s = \frac{24^2}{2 \times 9.8} = 29 \text{ (to 2 s.f.)}$$

The greatest height reached by the pebble above the point of projection is 29 m.

4 b $u = 24,\ a = -9.8,\ v = 0,\ t = ?$

$v = u + at$
$0 = 24 - 9.8t$
$t = \frac{24}{9.8} = 2.4$ (to 2 s.f.)

The time taken to reach the greatest height is 2.4 s.

5 a Take upwards as the positive direction.

$u = 18,\ a = -9.8,\ s = 15,\ v = ?$

$v^2 = u^2 + 2as = 18^2 - 2\times 9.8\times 15 = 30$
$v = \sqrt{30} = \pm 5.5$ (to 2 s.f.)

The speed of the ball when it is 15 m above its point of projection is 5.5 m s^{-1}.

b $u = 18,\ a = -9.8,\ s = -4,\ v = ?$

$v^2 = u^2 + 2as = 18^2 + 2\times(-9.8)\times(-4) = 324 + 78.4 = 402.4$
$v = -\sqrt{402.2} = -20$ (to 2 s.f.)

The speed with which the ball hits the ground is 20 m s^{-1}.

6 a Take downwards as the positive direction.

$s = 80,\ u = 4,\ a = 9.8,\ v = ?$

$v^2 = u^2 + 2as$
$= 4^2 + 2\times 9.8\times 80 = 1584$
$v = \sqrt{1584} = 40$ (to 2 s.f.)

The speed with which P hits the ground is 40 m s^{-1}.

b $u = 4,\ a = 9.8,\ v = \sqrt{1584},\ t = ?$

$v = u + at$
$\sqrt{1584} = 4 + 9.8t$
$$t = \frac{\sqrt{1584} - 4}{9.8} = 3.7 \text{ (to 2 s.f.)}$$

The time P takes to reach the ground is 3.7 s.

7 a Take upwards as the positive direction.

$v = -10,\ a = -9.8,\ t = 5,\ u = ?$

$$\begin{aligned} v &= u + at \\ -10 &= u - 9.8 \times 5 \\ u &= 9.8 \times 5 - 10 = 39 \end{aligned}$$

The speed of projection of P is 39 m s^{-1}.

b $u = 39,\ v = 0,\ a = -9.8,\ s = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0^2 &= 39^2 - 2 \times 9.8 \times s \\ s &= \frac{1521}{2 \times 9.8} = 78 \text{ (to 2 s.f.)} \end{aligned}$$

The greatest height above X attained by P during its motion is 78 m.

8 Take upwards as the positive direction.

$u = 21,\ t = 4.5,\ a = -9.8,\ s = ?$

$$s = ut + \tfrac{1}{2}at^2 = 21 \times 4.5 - 4.9 \times 4.5^2 = -4.7 \text{ (to 2 s.f.)}$$

The height above the ground from which the ball was thrown is 4.7 m.

9 Take upwards as the positive direction.

Find time when stone is instantaneously stationery:

$v = 0,\ u = 16,\ a = -9.8,\ t = ?$

$v = u + at$

$0 = 16 - 9.8t$

$t = \dfrac{16}{9.8} = 16.326... = 1.6$ s (to 1 d.p.)

So the stone is instantaneously stationary at 1.6 s

Find time of flight:

$s = -3,\ u = 16,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$-3 = 16t - 4.9t^2$

$4.9t^2 - 16t - 3 = 0$, so using the quadratic formula,

$$t = \frac{-(-16) \pm \sqrt{(-16)^2 - 4\times(4.9)\times(-3)}}{2\times(4.9)}$$

$t = 3.4431... = 3.4$ (to 1 d.p.) as we may discount the negative answer.

So the time of flight of the stone is 3.4 s.

Find speed when stone hits the ground:

$v = ?,\ u = 16,\ a = -9.8,\ t = 3.4431...$

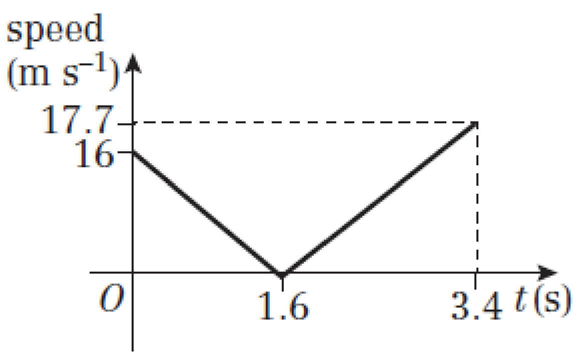
$v = u + at$

$v = 16 - 9.8 \times 3.4431...$

$v = -17.74... = -17.7$ ms^{-1} (to 1 d.p.)

So speed when stone hits the ground is 17.7 ms^{-1}

Sketch speed-time graph



10 Take upwards as the positive direction.

$u = 24.5,\ a = -9.8,\ s = 21,\ t = ?$

$s = ut + \frac{1}{2}at^2$
$21 = 24.5t - 4.9t^2$

$4.9t^2 - 24.5t + 21 = 0$

Using the quadratic formula,

$$t = \frac{-(-24.5) \pm \sqrt{(-24.5)^2 - 4 \times (4.9) \times (21)}}{2 \times (4.9)}$$

$= 1.1 \text{ or } 3.9$

The difference between these times is
$(3.9 - 1.1)\text{ s} = 2.8\text{ s}$

The total time for which the particle is 21 m or more above its point of projection is 2.8 s.

11 a Take upwards as the positive direction.

$v = \frac{1}{3}u,\ a = -9.8,\ t = 2,\ u = ?$

$v = u + at$
$\frac{1}{3}u = u - 9.8 \times 2$
$\frac{2}{3}u = 19.6 \Rightarrow u = \frac{3}{2} \times 19.6 = 29.4$

$u = 29$ (to 2 s.f.)

b $u = 29.4,\ s = 0,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$
$0 = 29.4t - 4.9t^2 = t(29.4 - 4.9t),\ t \neq 0$
$t = \frac{29.4}{4.9} = 6$

The time from the instant that the particle leaves O to the instant that it returns to O is 6 s.

12 For A, take downwards as the positive direction, $s_A = ut + \frac{1}{2}at^2 = 5t + 4.9t^2$

For B, take upwards as the positive direction, $s_B = ut + \frac{1}{2}at^2 = 18t - 4.9t^2$

$s_A + s_B = 46$

$(5t + 4.9t^2) + (18t - 4.9t^2) = 46$

$23t = 46 \Rightarrow t = 2$

Substitute $t = 2$ into $s_A = 5t + 4.9t^2$

$s_A = 5 \times 2 + 4.9 \times 2^2 = 29.6 = 30$ (to 2 s.f.)

The distance of the point where A and B collide from the point where A was thrown is 30 m.

13 a Find the speed, u_1 say, immediately before the ball strikes the floor.

$u = 0, \ a = 9.8, \ s = 10, \ v = u_1$
$v^2 = u^2 + 2as$
$u_1^2 = 0^2 + 2 \times 9.8 \times 10 = 196$
$u_1 = \sqrt{196} = 14$

The speed of the first rebound, u_2 say, is given by

$u_2 = \frac{3}{4}u_1 = \frac{3}{4} \times 14 = 10.5$

Find the maximum height, h_1 say, reached after the first rebound.

$u = 10.5, \ v = 0, \ a = -9.8, \ s = h_1$

$v^2 = u^2 + 2as$
$0^2 = 10.5^2 - 2 \times 9.8 \times h_1$

13 a $h_1 = \dfrac{10.5^2}{2\times 9.8} = 5.6$ (to 2 s.f.)

The greatest height above the floor reached by the ball the first time it rebounds is 5.6 m.

b Immediately before the ball strikes the floor for the second time, its speed is again $u_2 = 10.5$ by symmetry. The speed of the second rebound, u_3 say, is given by

$$u_3 = \tfrac{3}{4}u_2 = \tfrac{3}{4}\times 10.5 = 7.875$$

Find the maximum height, h_2 say, reached after the second rebound.

$u = 7.875,\ v = 0,\ a = -9.8,\ s = h_2$

$$v^2 = u^2 + 2as$$
$$0^2 = 7.875^2 - 2\times 9.8\times h_2$$
$$h_2 = \frac{7.875^2}{2\times 9.8} = 3.2 \text{ (to 2 s.f.)}$$

The greatest height above the floor reached by the ball the second time it rebounds is 3.2 m.

Challenge

1 a Take upwards as the positive direction.

For P, $s = ut + \frac{1}{2}at^2$ gives $s_P = 12t - 4.9t^2$

For Q, $s = ut + \frac{1}{2}at^2$

Q has been moving for 1 less second than P, so

$$s_Q = 20(t-1) - 4.9(t-1)^2$$

At the point of collision $s_P = s_Q$

$$\begin{aligned} 12t - 4.9t^2 &= 20(t-1) - 4.9(t-1)^2 \\ &= 20t - 20 - 4.9t^2 + 9.8t - 4.9 \end{aligned}$$

$24.9 = 17.8t \Rightarrow t = \frac{24.9}{17.8} = 1.4$ (to 2 s.f.)

The time between the instant when P is projected and the instant when P and Q collide is 1.4 s.

Challenge

1 b Substitute t into $s_P = 12t - 4.9t^2$ from part **a**

$s_P = 12t - 4.9t^2 \approx 12\times1.4 - 4.9\times1.4^2 = 7.2$ (to 2 s.f.)

The distance of the point where P and Q collide from O is 7.2 m.

2 Take downwards as positive.

For 1st stone: $u = 0$, $t = t_1$, $a = 9.8$, $s = h$

$s = ut + \frac{1}{2}at^2$

$h = 0\times t_1 + \frac{1}{2}\times9.8\times t_1^2 = 4.9t_1^2$

For 2nd stone: $u = 25$, $t = t_1 - 2$, $a = 9.8$, $s = h$

$s = ut + \frac{1}{2}at^2$

$$\begin{aligned} h &= 25(t_1 - 2) + \tfrac{1}{2}(9.8\times(t_1 - 2)^2) \\ &= 25t_1 - 50 + 4.9\times(t_1^2 - 4t_1 + 4)) \\ &= 25t_1 - 50 + 4.9t_1^2 - 19.6t_1 + 19.6 \\ &= 4.9t_1^2 + 5.4t_1 - 30.4 \end{aligned}$$

Substituting for h from information for first stone:

$$\begin{aligned} 4.9t_1^2 &= 4.9t_1^2 + 5.4t_1 - 30.4 \\ 30.4 &= 5.4t_1 \\ t_1 &= \frac{30.4}{5.4} = 5.629 \end{aligned}$$

Putting this value into equation for first stone:

$h = 4.9\times5.629^2 = 4.9\times31.69 = 155$ (to 3 s.f.)

The height of the building is 155 m.