Review Exercise 2

1 a For the first 3 s the cyclist is moving with constant acceleration.

b For the remaining 4 s the cyclist is moving with constant speed.

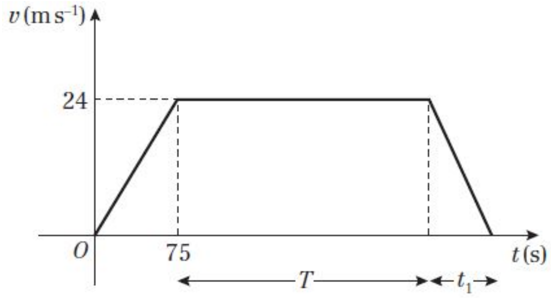
c area = trapezium + rectangle

$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$

$= 10.5 + 20 = 30.5$

The distance travelled by the cyclist is 30.5 m

2 a



b Let time for which the train decelerates be t_1 s.

While decelerating

area $= \frac{1}{2}$ base × height

$600 = \frac{1}{2}t_1 \times 24$

$t_1 = \frac{1200}{24} = 50$

Acceleration is represented by the gradient.

$a = -\frac{24}{t_1} = -\frac{24}{50} = -0.48$

The deceleration is 0.48 m s^{-2}

c For the whole journey

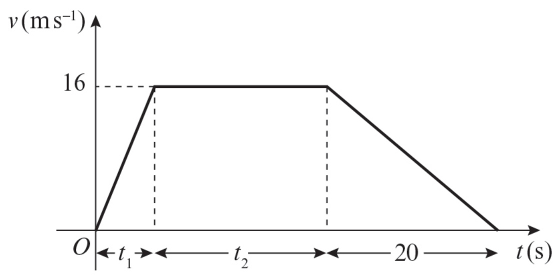
$s = \frac{1}{2}(a+b)h$

$7500 = \frac{1}{2}(T + T + 125)\times 24$

$$T = \frac{7500-1500}{24} = 250$$

d Total time is $(75 + T + t_1)\text{ s} = (75 + 250 + 50)\text{ s} = 375$ s

3 a



3 a Let the time for which the train accelerates be t_1 s and the time for which it travels at a constant speed be t_2 s.

During acceleration

$v = u + at$

$16 = 0 + 0.4t_1 \Rightarrow t_1 = \frac{16}{0.4} = 40$

At constant speed

$2000 = 16 \times t_2 \Rightarrow t_2 = \frac{2000}{16} = 125$

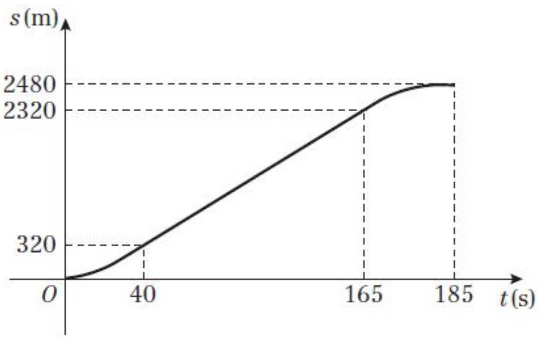
The total time is $(t_1 + t_2 + 20)\,\text{s} = (40 + 125 + 20)\,\text{s} = 185\,\text{s}$

b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(125 + 185) \times 16 = 2\,480$

$AB = 2\,480\,\text{m}$

c



4 a Taking the upwards direction as positive.

$s = 40,\ v = 0,\ a = -9.8,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 - 2 \times 9.8 \times 40$

$u^2 = 784 \Rightarrow u = 28$

The speed of projection is 28 m s^{-1}

b $s = 0,\ u = 28,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 28t - 4.9t^2 = t(28 - 4.9t)$

$t = 0,\ t = \frac{28}{4.9} = 5.714...$

The time taken to return to A is 5.7 s (2 s.f.)

5 Find the speed of projection.
Taking the upwards direction as positive.

$v = 0,\ t = 3,\ a = -9.8,\ u = ?$

5 $v = u + at$

$0 = u - 9.8 \times 3 \Rightarrow u = 29.4$

$s = 39.2,\ u = 29.4,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$39.2 = 29.4t - 4.9t^2$

$4.9t^2 - 29.4t + 39.2 = 0$

Dividing all terms by 4.9

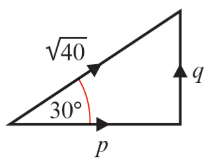
$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$

The ball is 39.2 m above its point of projection when $t = 2$ or when $t = 4$

6



$$\tan 30 = \frac{1}{\sqrt{3}} = \frac{q}{p}$$

$$\therefore q = \frac{p}{\sqrt{3}}$$

$$40 = p^2 + q^2$$

$$p^2 + \frac{p^2}{3} = 40$$

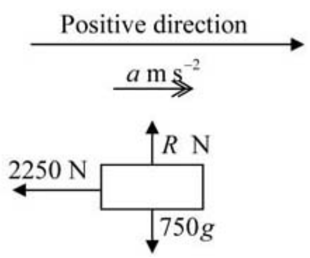
$$\frac{4}{3}p^2 = 40$$

$$p^2 = 30$$

$$\therefore q^2 = 40 - 30 = 10$$

The values are: $p = \sqrt{30}$ and $q = \sqrt{10}$.

7



$F = ma$

$R(\rightarrow)\ \ -2250 = 750a$

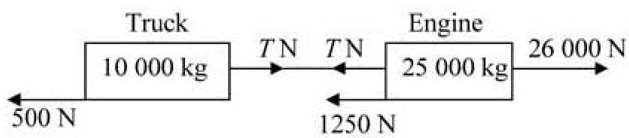
$a = -\frac{2250}{750} = -3$

7 $u = 25, v = 15, a = -3, s = ?$

$$v^2 = u^2 + 2as$$
$$15^2 = 25^2 - 6s$$
$$s = \frac{25^2 - 15^2}{6} = \frac{400}{6} = 66\tfrac{2}{3}$$

The distance travelled by the car as its speed is reduced is $66\frac{2}{3}$ m**.**

8



a The resistance on the engine is $25 \times 50 = 1250$ N
The resistance on the truck is $10 \times 50 = 500$ N
For the whole system, the engine and truck

$R(\rightarrow)$ $\quad F = ma$

$$26\,000 - 1250 - 500 = 35\,000a$$
$$a = \frac{26\,000 - 1250 - 500}{35\,000} = \frac{97}{140} = 0.6928...$$

The acceleration of the engine and truck is $0.693\,\text{m}\,\text{s}^{-2}$ (3 s.f.)

b For the truck alone

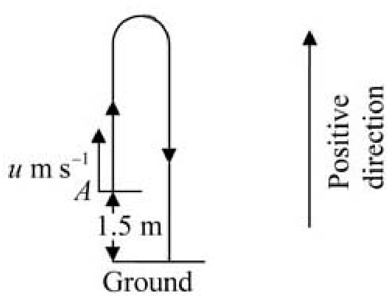
$$F = ma$$
$$T - 500 = 10\,000a$$
$$T = 500 + 10\,000 \times 0.6928... = 7428.57...$$

The tension in the coupling is 7430 N (3 s.f.)

c **i** Treating the engine and truck as particles allows us to assume the weight acts from the centre of mass of each object, and ignore wind resistance and rotational forces.

ii By assuming the coupling is a light horizontal rod, we treat it as if it had no mass and therefore can assume it not only stays straight but that it has no weight and the tension is constant along the entire length.

9



9 a From A to the greatest height, taking upwards as positive.

$v = 0,\ a = -9.8,\ s = 25.6,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

b $u = 22.4,\ s = -1.5,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2$

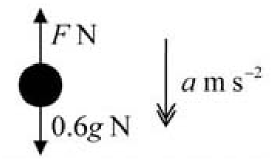
$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

$4.9T^2 - 22.4T - 1.5 = 0$

$$T = \frac{22.4 + \sqrt{(-22.4)^2 - 4 \times 4.9 \times -1.5}}{2 \times 4.9}$$

$= 4.637... = 4.64$ (3 s.f.)

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ s = -1.5,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ v = 0,\ s = 0.025,\ a = ?$

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

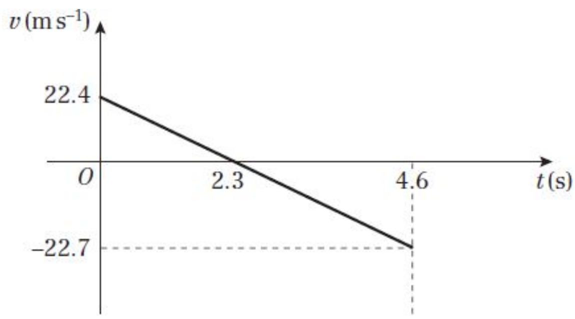
$a = -\frac{531.16}{0.05} = -10\,623.2$

$F = ma$

$0.6g - F = 0.6 \times (-10\,623.2)$

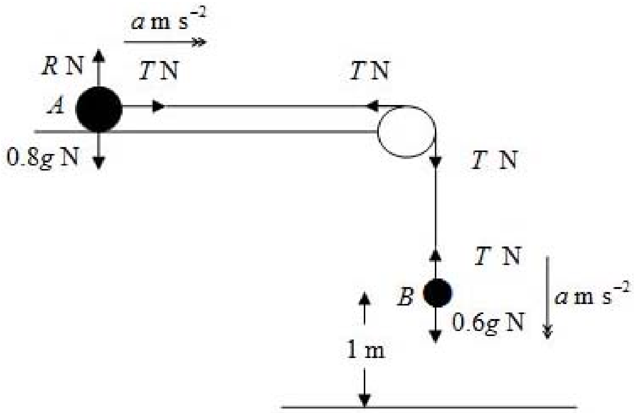
$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.)

d



9 e Consider air resistance during motion under gravity.

10



a For A

$R(\rightarrow)$ $\quad T = 0.8a \quad$ **(1)**

For B

$R(\downarrow)$ $\quad 0.6g - T = 0.6a \quad$ **(2)**

(1) + **(2)**

$0.6g = 1.4a$

$$a = \frac{0.6 \times 9.8}{1.4} = 4.2$$

The acceleration of A is 4.2 m s^{-2}

b Substitute $a = 4.2$ into (1)

$T = 0.8 \times 4.2 = 3.36$

The tension in the string is 3.4 N (2 s.f.)

c $u = 0,\ a = 4.2,\ s = 1,\ v = ?$

$v^2 = u^2 + 2as$

$\quad = 0^2 + 2 \times 4.2 \times 1 = 8.4$

$v = \sqrt{8.4} = 2.898...$

The speed of B when it hits the ground is 2.9 m s^{-1} (2 s.f.)

d $u = 0,\ a = 4.2,\ s = 1,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$1 = 0 + 2.1t^2$

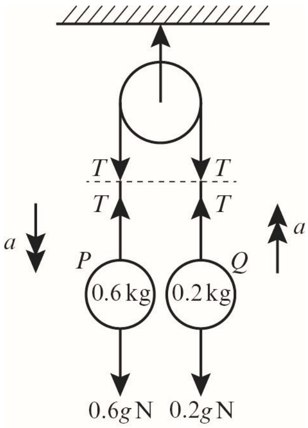
$t^2 = \frac{1}{2.1} \Rightarrow t = 0.690...$

The time taken for B to reach the ground is 0.69 s (2 s.f.)

e i Describing the string as ‘light’ means it has no mass (and therefore no weight).

10 e ii This fact allows us to assume that the tension is constant in all parts of the string.

11



a $F = ma$

For $P \downarrow$ positive: $0.6a = 0.6g - T$ **(1)**

For $Q \uparrow$ positive: $0.2a = T - 0.2g$ **(2)**

$3 \times$ (**2**): $0.6a = 3T - 0.6g$

Subtracting (**1**) from this: $0 = 4T - 1.2g$

$4T = 1.2g = 1.2 \times 9.8$

The tension in the string is 2.9 N (2 s.f.)

b (1) + (2): $0.8a = 0.4g$

$$a = \frac{g}{2} = \frac{9.8}{2}$$

The acceleration is 4.9 m s^{-2}.

c For P, before string breaks, taking up as positive:

$s = ?$, $u = 0$, $a = 4.9$, $t = 0.4$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = (0 \times 0.4) + \tfrac{1}{2}(4.9 \times 0.4^2)$$

$$= \tfrac{1}{2}(0.784)$$

$$= 0.392\,\text{m}$$

The total distance P has to fall is therefore $1 - 0.392 = 0.608$ m.

$v = u + at$

$v = 0 + (0.4 \times 4.9) = 1.96$ m s^{-1}

Before the string breaks, P is moving downwards at 1.96 m s^{-1}. After string breaks, taking down as positive, P moves under gravity.

$s = 0.608$, $u = 1.96$, $a = 9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$

$$0.608 = 1.96t + \tfrac{1}{2}(9.8 \times t^2)$$

$$0 = 4.9t^2 - 1.96t - 0.608$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

11 c $t = \dfrac{-1.96 \pm \sqrt{(-1.96)^2 - (4 \times 4.9 \times -0.608)}}{2 \times 4.9}$

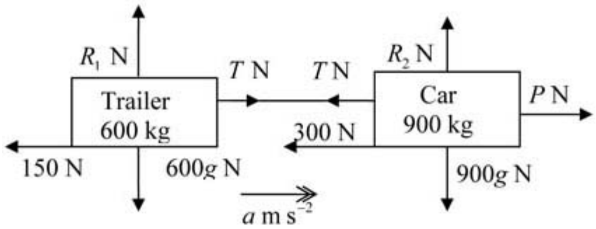
$= \dfrac{-1.96 \pm \sqrt{15.76}}{9.8}$

$= 0.205 \text{ s or } -0.605 \text{ s (3 d.p.)}$

Only positive answers are relevant in this context. $\therefore$ P hits the floor 0.21 s (2 s.f.) after the string breaks.

d This fact allows us to assume that the tension is constant in all parts of the string and that the acceleration of the two particles is the same.

12



a i For the whole system:

$$F = ma$$

$R(\rightarrow)$ $\quad P - 300 - 150 = 1500 \times 0.4$

$$P = 1050$$

The tractive force exerted by the engine of the car is 1050 N.

ii For the trailer alone:

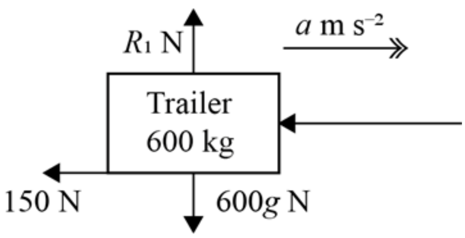
$$F = ma$$

$R(\rightarrow)$ $\quad T - 150 = 600 \times 0.4$

$$T = 390$$

The tension in the tow bar is 390 N.

b For the trailer alone:



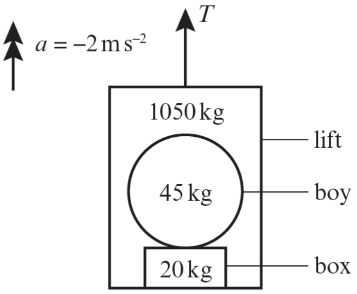
$$F = ma$$

$R(\rightarrow)$ $\quad 1650 + 150 = 600a$

$$a = -\tfrac{1800}{600} = -3$$

The greatest possible deceleration of the car is 3 m s^{-2}

13

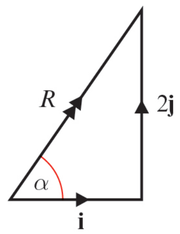


a $F = ma$
Taking up as positive:
$(1050 + 45 + 20) \times -2 = T - (1050 + 45 + 20)g$
$T = 1115(g - 2)$
$T = 1115 \times 7.8$
The tension in the cable is 8697 N.

b From Newton's third law of motion:
|Force exerted on boy by box| = |Force exerted on box by boy| = $|R_1|$
For the boy, taking up as positive:
$45 \times -2 = R_1 - 45g$
$R_1 = 45(g - 2)$
$R_1 = 45 \times 7.8$
The boy exerts a force of 351 N on the box.

c From Newton's third law of motion:
|Force exerted on box by lift| = |Force exerted on lift by box| = $|R_2|$
For the box, taking up as positive:
$20 \times -2 = R_2 - 20g - 351$
$R_2 = 351 + 20(g - 2)$
$R_2 = 351 + (20 \times 7.8) = 351 + 156$
The box exerts a force of 507 N on the lift.

14 a



Let the required angle be α.
Then $\tan\alpha = 2$
$\therefore \alpha = 63^{\circ}\,(2\text{ s.f.})$

b As $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{R}$
$(2\mathbf{i} + 3\mathbf{j}) + (\lambda\mathbf{i} + \mu\mathbf{j}) = k(\mathbf{i} + 2\mathbf{j})$

where k is a constant.
$\therefore 2 + \lambda = k$ and $3 + \mu = 2k$ *

Eliminate k from these two equations.

14 b Then $2(2+\lambda)=3+\mu$

$\therefore 2\lambda-\mu+1=0$

c If $\mathbf{F}_2$ is parallel to $\mathbf{j}$ then $\lambda=0$

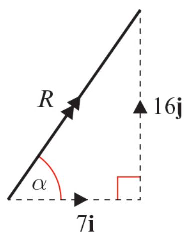
Substituting $\lambda=0$ into * gives

$\mu=1$ and $k=2$

$\therefore \mathbf{R}=2\mathbf{i}+4\mathbf{j}$

$$\therefore |\mathbf{R}|=\sqrt{2^2+4^2}$$
$$=\sqrt{20}$$
$$=4.47 \text{ (3 s.f.)}$$

15



a The magnitude of

$$\mathbf{R}=\sqrt{7^2+16^2}$$
$$=17.5 \text{ (1 d.p.)}$$

b $\tan\alpha=\frac{16}{7}$

$$\alpha=\tan^{-1}\left(\tfrac{16}{7}\right)$$
$$=66^\circ \text{ (nearest degree)}$$

c Let $\mathbf{P}=\lambda(\mathbf{i}+4\mathbf{j})$ and $\mathbf{Q}=\mu(\mathbf{i}+\mathbf{j})$

As $\mathbf{P}+\mathbf{Q}=\mathbf{R}$

$\therefore \lambda(\mathbf{i}+4\mathbf{j})+\mu(\mathbf{i}+\mathbf{j})=(7\mathbf{i}+16\mathbf{j})$

Equating $\mathbf{i}$ components

$\lambda+\mu=7$ **(1)**

Equating $\mathbf{j}$ components

$4\lambda+\mu=16$ **(2)**

Subtract **(2)** − **(1)**

$3\lambda=9$

$\therefore \lambda=3$

15 Substitute into equation (**1**)

$\therefore 3+\mu=7$

$\therefore \mu=4$

$\therefore \mathbf{P}=3(\mathbf{i}+4\mathbf{j})=3\mathbf{i}+12\mathbf{j}$ and $\mathbf{Q}=4(\mathbf{i}+\mathbf{j})=4\mathbf{i}+4\mathbf{j}$

16 $a=5-2t$

$v=\int a\mathrm{d}t=\int(5-2t)\mathrm{d}t$

$=5t-t^2+C$

When $t=0$, $v=6$

$6=0-0+C\Rightarrow C=6$

Hence

$v=6+5t-t^2$

When P is at rest

$0=6+5t-t^2$

$t^2-5t-6=(t-6)(t+1)=0$

$t=6,-1$

$t>0$

$\therefore t=6$

P is at rest at $t=6$ s

17 $v=6t-2t^2$

a Maximum value of velocity occurs when $a=0$

$a=\dfrac{dv}{dt}=6-4t$

Maximum velocity occurs at $t=\dfrac{3}{2}$ s

$v=\left(6\times\dfrac{3}{2}\right)-2\left(\dfrac{3}{2}\right)^2$

$v=9-\dfrac{9}{2}=\dfrac{9}{2}$

The maximum velocity is 4.5 m s^{-1}.

b When P returns to O, $s=0$

$s=\int vdt=\int 6t-2t^2dt$

$s=3t^2-\dfrac{2}{3}t^3+c$

At $t=0$, $s=0$ so $c=0$

17 $0 = t^2\left(3 - \frac{2}{3}t\right)$

$t = 0$ or $\frac{2}{3}t = 3$

P returns to O after 4.5 s.

18 $v = 3t^2 - 8t + 5$

a When the particle is at rest, $v = 0$

$0 = 3t^2 - 8t + 5$

$0 = 3\left(t^2 - \frac{8}{3}t + \frac{5}{3}\right)$

$0 = 3\left(t - \frac{3}{3}\right)\left(t - \frac{5}{3}\right)$

(or by using quadratic equation formula)

P is at rest at 1 s and $\frac{5}{3}$ s.

b $a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 8t + 5)$

$a = 6t - 8$

$t = 4$

$a = (6 \times 4) - 8$

After 4 s, the acceleration of P is 16 m s^{-2}.

c Distance travelled in third second $= s_3$

$s_3 = \int_2^3 v\,dt = \int_2^3 3t^2 - 8t + 5\,dt$

$s_3 = \left[t^3 - 4t^2 + 5t\right]_2^3$

$s_3 = [27 - 36 + 15] - [8 - 16 + 10]$

$s_3 = 6 - 2$

The distance travelled in the third second is 4 m.

19 $v = 6t - 2t^{\frac{3}{2}}$

a $a = \frac{dv}{dt}$

$a = 6 - 3t^{\frac{1}{2}}$

b At $t = 0$, $s = 0$

$s = \int v\,dt = \int 6t - 2t^{\frac{3}{2}}\,dt$

$s = 3t^2 - \frac{4}{5}t^{\frac{5}{2}} + c$

At $t = 0$, $s = 0$, $c = 0$

So $s = 3t^2 - \frac{4}{5}t^{\frac{5}{2}}$

Challenge

1 $t_1 + t_2 + t_3 = 7 \times 60 = 420$
$3t_1 = 4t_3$
$t_3 = 0.75t_1$

Considering time t_1

$$s = \left(\frac{u+v}{2}\right)t_1$$

$$1750 = \left(\frac{0+v}{2}\right)t_1$$

$$v = \frac{3500}{t_1}$$

Considering time t_2
$s_2 = vt_2$

$$17\,500 = \frac{3500}{t_1}t_2$$

$$t_2 = 5t_1$$

Considering total time:
$t_1 + 5t_1 + 0.75\ t_1 = 420$

$$t_1 = \frac{420}{6.75} = 62.22\ \text{s}$$

$\therefore t_2 = 311.11$ s
& $t_3 = 46.67$ s

Distance travelled during time t_3 is s_3

$$s_3 = \left(\frac{u+v}{2}\right)t_3$$

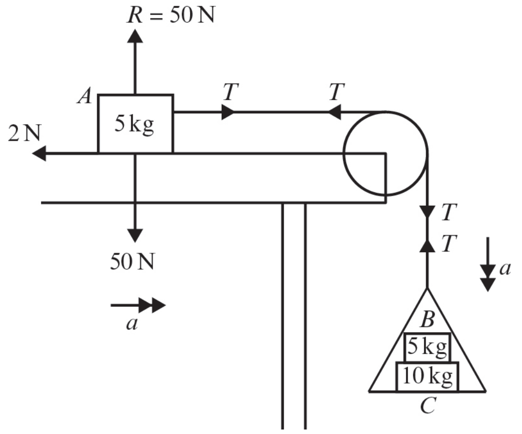
$$u = \frac{3500}{t_1} = \frac{3500}{62.22} = 56.25\ ,\ v = 0,\ t = 46.67$$

$$s_3 = \left(\frac{56.25+0}{2}\right)46.67$$

$$s_3 = 28.125 \times 46.67 = 1312.6$$

Total distance = 1750 + 17 500 + 1312.6
The distance between the two stations is 20.6 km (3 s.f.).

2



a Considering A, → positive: $T - 2 = 5a$
Considering entire pan, ↓ positive: $(5 + 10)g - T = 15a$
So $150 - T = 15a$

Adding these gives:
$148 = 20a$
The acceleration of the pan is 7.4 m s^{-2}.

b Substituting this value into the first equation gives:
$T - 2 = 5 \times 7.4 = 37$
The tension in the string is 39 N.

c Block C exerts a normal reaction force R on block B.
Considering block B only, ↓ positive:
$5g - R = 5a$
$50 - R = 37$
Block C exerts a force of 13 N on block B.

d The force the string exerts on the pulley has two perpendicular components, each of magnitude T.
The magnitude of the total force, F, is therefore given by:
$F^2 = T^2 + T^2$
$F = \sqrt{39^2 + 39^2} = \sqrt{3042}$
The string exerts a force of magnitude 55 N (2 s.f.) on the pulley.

e The fact that the string is inextensible allows us to assume that the tension is constant in all parts of the string and that the acceleration of Block A and the scale pan are the same.