The normal distribution 3A

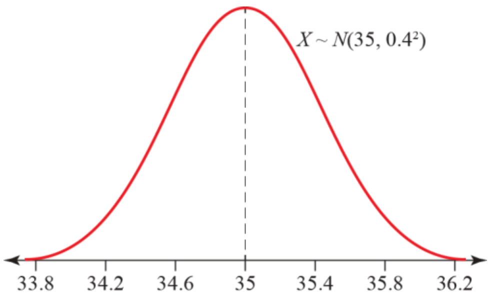
1 **a** Continuous – lengths can take any value.

b Discrete – scores can only take certain values.

c Continuous – masses can take any value.

d Discrete – shoe sizes can only take certain values.

2 Since the mean is 35 mm, the distribution should be symmetrical about this value. Since the standard deviation is 0.4 mm, 68% of the data should lie in the range 34.6 mm to 35.4 mm and 95% of the data should lie in the range 34.2 mm to 35.8 mm.



3 One of the key features of normal distributions is that they are symmetrical about the mean (which is equal to the mode and the median). This curve shows that the bank employees' incomes are not equally distributed either side of its peak (the modal income), so the normal distribution is not a suitable model.

4 **a** Since $\sigma^2 = 16$, the standard deviation is $\sqrt{16} = 4$ cm and the mean is 120 cm.
68% of the pupils are expected to have an armspan within one standard deviation of the mean, i.e. within the interval 116 cm to 124 cm.

b The given interval, 112 cm to 128 cm, includes all of the pupils whose armspans are up to two standard deviations either side of the mean. Therefore 95% of the pupils can be expected to have an armspan within this range.

5 If 68% of the adders have a length between 93 cm (100 cm − 7 cm) and 107 cm (100 cm + 7 cm), then the standard deviation is 7 cm. Therefore the variance, σ^2, is $7^2 = 49$.

6 Since 95% of the data should lie within two standard deviations of the mean, 2.5% of the data should be two standard deviations or more below the mean and 2.5% of the data should be two standard deviations or more above the mean. Since 2.5% of the dormice weigh more than 70 grams, this means that 70 grams is two standard deviations above the mean. The standard deviation is 5 grams (the square root of the variance) so the mean is 60 grams.

7 In a normal distribution, 68% of the data lies within one standard deviation, σ, of the mean, μ, and 32% lies outside of this range. Therefore 16% of the data lies below $\sigma - \mu$, and 16% lies above $\sigma + \mu$. Also, 95% of the data lies between $\mu - 2\sigma$ and $\mu + 2\sigma$. Therefore 2.5% lies below $\mu - 2\sigma$ and 2.5% lies above $\mu + 2\sigma$.

The question states that 84% of the pigs weigh more than 52 kg, so 16% weigh less than 52 kg. Hence $52 = \mu - \sigma$. Also, 97.5% of the pigs weigh more than 47.5 kg, so 2.5% weigh less than 47.5 kg and $47.5 = \mu - 2\sigma$. From these two equations, $\sigma = 52 - 47.5 = 4.5$ kg and so $\sigma^2 = (4.5)^2 = 20.25$.
Hence $\mu = 52 + \sigma = 52 + 4.5 = 56.5$ kg.

8 a Since the normal distribution is symmetrical and the mean is equal to the median and the mode, half the scores should be above 45 and half should be below. Therefore $\text{P}(S > 45) = 0.5$.

b Since the data within one standard deviation of the mean should be 68% of the sample, $\text{P}(30 < S < 60) = 0.68$.

c Since the data within two standard deviations of the mean should be 95% of the sample, $\text{P}(15 < S < 75) = 0.95$.

d Alexia is incorrect: although $\text{P}(X > 100) > 0$, the value is very small as 100 as more than three standard deviations from the mean, so the model as a whole is still reasonable.

9 a The mean of the normal distribution is where the highest point on the curve appears. From the sketch, this is around 36 cm.

b The points of inflection of the normal occur at $\mu + \sigma$ and $\mu - \sigma$. From the sketch, these points lie somewhere in the intervals [33, 34] and [38, 39]. Since the mean is around 36 cm, this means that the standard deviation should be somewhere between 2 cm and 3 cm.