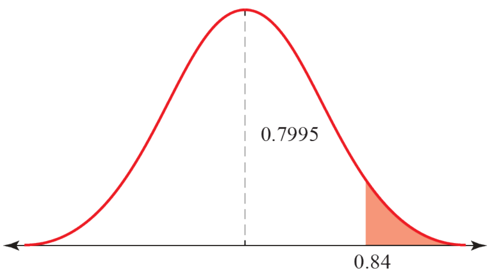
The normal distribution 3D

1 Use the Normal CD function on your calculator, with $\mu = 0$, $\sigma = 1$ and a small value for the lower limit, e.g. −10.

a $P(Z < 2.12) = 0.98299... = 0.9830$ (4 d.p.)

b $P(Z < 1.36) = 0.91308... = 0.9131$ (4 d.p.)

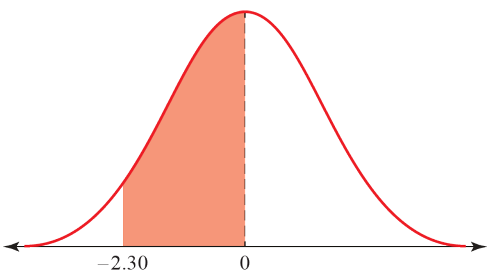
c



$P(Z > 0.84)$
$= 1 - P(Z < 0.84)$
$= 1 - 0.79954...$
$= 0.20045... = 0.2005$ (4 d.p.)

d $P(Z < -0.38) = 0.35197... = 0.3520$ (4 d.p.)

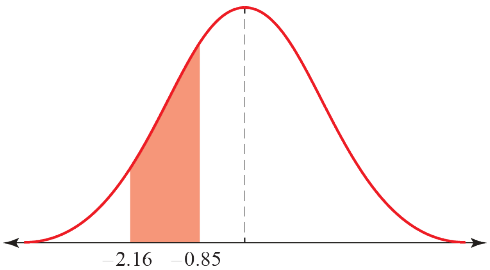
e



$P(-2.30 < Z < 0)$
$= 0.5 - P(Z < -2.30)$
$= 0.5 - 0.1072...$
$= 0.48929... = 0.4893$ (4 d.p.)

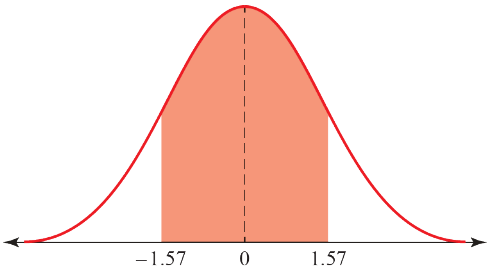
f $P(Z < -1.63) = 0.05155... = 0.0516$ (4 d.p.)

1 g



$P(-2.16 < Z < -0.85)$
$= P(Z < -0.85) - P(Z < -2.16)$
$= 0.19766... - 0.01538...$
$= 0.18227... = 0.1823$ (4 d.p.)

h



$P(-1.57 < Z < 1.57)$
$= 2 \times (0.5 - P(Z < -1.57))$
$= 2 \times (0.5 - 0.05820...)$
$= 2 \times 0.44179...$
$= 0.88358... = 0.8836$ (4 d.p.)

2 Use the inverse normal distribution function on your calculator, with $\mu = 0$ and $\sigma = 1$.

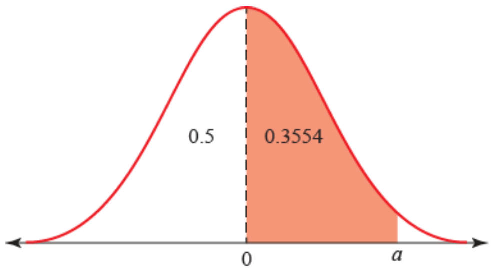
a $P(Z < a) = 0.9082 \Rightarrow a = 1.32975... = 1.3298$ (4 d.p.)

b $P(Z > a) = 0.0314$
$\Rightarrow P(Z < a) = 0.9686$
$\Rightarrow a = 1.86060... = 1.8606$ (4 d.p.)

c $P(Z > a) = 0.15$
$\Rightarrow P(Z < a) = 0.85$
$\Rightarrow a = 1.03643... = 1.0364$ (4 d.p.)
(Alternatively, use the table of percentage points with $p = 0.15 \Rightarrow a = 1.0364$)

d $P(Z > a) = 0.95$
$\Rightarrow P(Z < a) = 0.05$
$\Rightarrow a = -1.64485... = -1.6449$ (4 d.p.)
(Alternatively, use the table of percentage points with $p = 0.05 \Rightarrow -a = 1.6449 \Rightarrow a = -1.6449$)

2 **e**

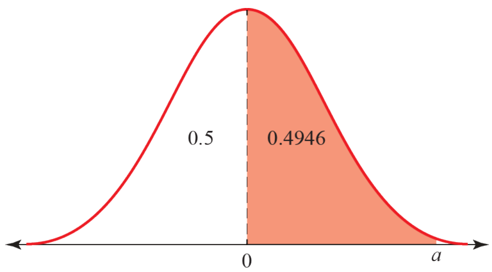


$\text{P}(0 < Z < a) = 0.3554$

$\Rightarrow \text{P}(Z < a) = 0.8554$

$\Rightarrow a = 1.05987... = 1.0599$ (4 d.p.)

f

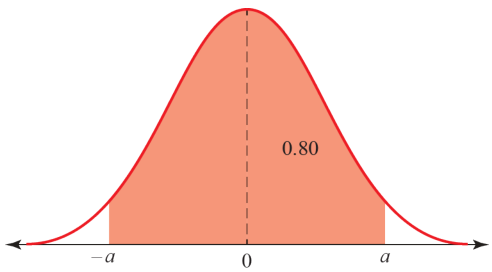


$\text{P}(0 < Z < a) = 0.4946$

$\Rightarrow \text{P}(Z < a) = 0.9946$

$\Rightarrow a = 2.54910... = 2.5491$ (4 d.p.)

g



$\text{P}(-a < Z < a) = 0.80$

$\Rightarrow \text{P}(-a < Z < 0) = 0.40$

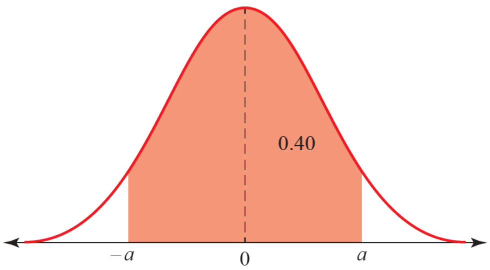
$\Rightarrow \text{P}(-a < Z) = 0.10$

$\Rightarrow -a = -1.28155...$

$\Rightarrow a = 1.2816$ (4 d.p.)

(Alternatively, use the table of percentage points with $p = 0.10 \Rightarrow a = 1.2816$)

2 h



$P(-a < Z < a) = 0.40$
$\Rightarrow P(-a < Z < 0) = 0.20$
$\Rightarrow P(-a < Z) = 0.30$
$\Rightarrow -a = -0.52440...$
$\Rightarrow a = 0.5244$ (4 d.p.)
(Alternatively, use the table of percentage points with $p = 0.30 \Rightarrow a = 0.5244$)

3 a $x = 0.8 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{0.8-0.8}{0.05} = 0$

b $x = 0.792 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{0.792-0.8}{0.05} = -0.16$

c $x = 0.81 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{0.81-0.8}{0.05} = 0.2$

d $x = 0.837 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{0.837-0.8}{0.05} = 0.74$

4 a $x = 154 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{154-154}{12} = 0 \Rightarrow P(X < 154) = \Phi(0)$

b $x = 160 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{160-154}{12} = 0.5 \Rightarrow P(X < 160) = \Phi(0.5)$

c $x = 151 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{151-154}{12} = -0.25 \Rightarrow P(X > 151) = 1 - P(X < 151) = 1 - \Phi(-0.25)$

d $x = 140 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{140-154}{12} = -\dfrac{7}{6}$

$x = 155 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{155-154}{12} = \dfrac{1}{12}$

$\Rightarrow P(140 < X < 155) = P(X < 155) - P(X < 140) = \Phi\left(\dfrac{1}{12}\right) - \Phi\left(-\dfrac{7}{6}\right)$

5 a $P(Z > z) = 0.025 \Rightarrow p = 0.025$
Using the percentage points table, $p = 0.025 \Rightarrow z = 1.96$

5 b Using the formula $z = \frac{x-\mu}{\sigma}$:

$$1.96 = \frac{x-80}{4}$$
$$x - 80 = 4 \times 1.96$$
$$x = 80 + 7.84$$
$$= 87.84$$

A score of 87.8 (3 s.f.) is needed to get on the programme.

6 a From the percentage points table, $p = 0.15 \Rightarrow z = 1.0364$

Therefore $P(Z > 1.0364) = 0.15$, hence $P(Z < -1.0364) = 0.15$, so $z = -1.0364$

b Using the formula $z = \frac{x-\mu}{\sigma}$:

$$-1.0364 = \frac{x-57}{2}$$
$$x - 57 = 2 \times (-1.0364)$$
$$x = 57 - 2.0728$$
$$= 54.9272$$

The size of a ‘petite’ hat is 54.9 cm (3 s.f.).

7 a The 90th percentile corresponds to $p = 0.1$.

From the percentage points table, $p = 0.10 \Rightarrow z = 1.2816$

By the symmetry of the normal distribution, the 10th percentile is at $z = -1.2816$

So the 10% to 90% interpercentile range corresponds to $-1.2816 < z < 1.2816$

b A ‘standard’ light bulb should have a range of life within the above range, but for $N(1175, 56)$.

Using the formula $z = \frac{x-\mu}{\sigma}$ with $z = -1.2816$:

$$-1.2816 = \frac{x-1175}{56}$$
$$x - 1175 = 56 \times (-1.2816)$$
$$x = 1175 - 71.7696$$
$$= 1103.2304$$

Similarly, for $z = 1.2816$, $x = 1175 + 71.7696 = 1246.7696$.

So the range of life for a ‘standard’ bulb is 1103 to 1247 hours.