Forces and friction 5A

1 a i $12\cos 20° = 11.3\text{N}$ (3s.f.)

ii $12\cos 70° = 12\sin 20°$
$= 4.10\text{ N}$ (3s.f.)

iii (11.3**i** + 4.10**j**) N

b i $5\cos 90° = 0\text{N}$

ii $-5\cos 0° = 5\cos 180°$
$= -5\text{ N}$

iii − 5**j** N

c i $-8\cos 50° = -5.14\text{N}$ (3 s.f.)

ii $8\cos 40° = 6.13\text{N}$ (3 s.f.)

iii (–5.14**i** + 6.13**j**) N

d i $-6\cos 50° = -3.86\text{N}$ (3 s.f.)

ii $-6\cos 40° = -4.60\text{N}$ (3 s.f.)

iii (–3.86**i** – 4.60**j**) N

2 a i $8\cos 60° - 6 = -2\text{N}$

ii $8\cos 30° - 0 = 6.93\text{N}$ (3 s.f.)

b i $6\cos 40° + 5\cos 45° = 8.13\text{N}$ (3 s.f.)

ii $10 + 6\cos 50° - 5\cos 45° = 10.3\text{N}$ (3 s.f.)

c i $P\cos\alpha + Q - R\cos(90° - \beta) = P\cos\alpha + Q - R\sin\beta$

ii $P\cos(90° - \alpha) - R\cos\beta = P\sin\alpha - R\cos\beta$

3 a Using the cosine rule:

$R^2 = 25^2 + 35^2 - (2 \times 25 \times 35 \cos 80^\circ)$

$R^2 = 1850 - 303.88...$

$R = 39.320...$

Using the sine rule:

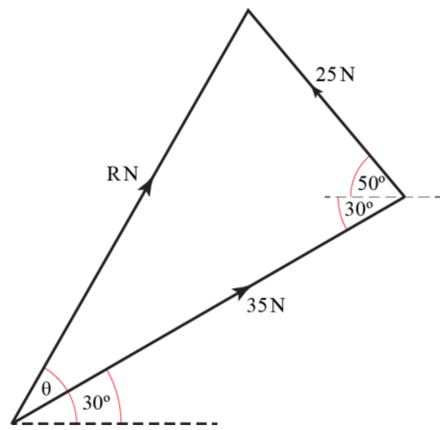
$\frac{\sin(\theta - 30^\circ)}{25} = \frac{\sin 80^\circ}{39.320...}$

$\sin(\theta - 30^\circ) = \frac{25 \sin 80^\circ}{39.320...}$

$(\theta - 30^\circ) = 38.765...^\circ$

$\theta = 68.765...^\circ$

The resultant force has a magnitude of 39.3 N (3s.f.) and acts at 68.8° above the horizontal (3s.f.).



b Using the cosine rule:

$R^2 = 20^2 + 15^2 - (2 \times 20 \times 15 \cos 105^\circ)$

$R^2 = 780.29...$

$R = 27.933...$

Using the sine rule:

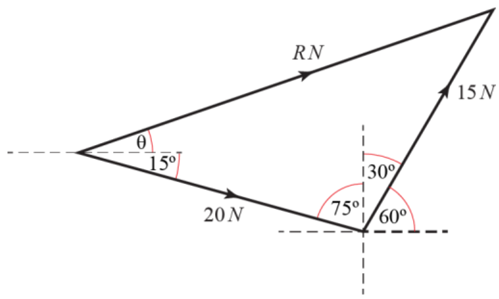
$\frac{\sin(15^\circ + \theta)}{15} = \frac{\sin 105^\circ}{27.933...}$

$\sin(15^\circ + \theta) = \frac{15 \sin 105^\circ}{27.933...}$

$(15^\circ + \theta) = 31.244...^\circ$

$\theta = 16.244...^\circ$

The resultant force has a magnitude of 27.9 N (3s.f.) and acts at 16.2° above the horizontal (3s.f.).



c Using the cosine rule then the sine rule, as before:

$R^2 = 5^2 + 2^2 - (2 \times 5 \times 2 \cos 5^\circ)$

$R^2 = 9.0761...$

$R = 3.0126...$

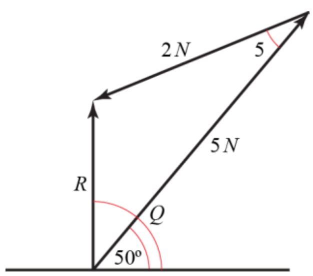
$\frac{\sin(\theta - 50^\circ)}{2} = \frac{\sin 5^\circ}{3.0126...}$

$\sin(\theta - 50^\circ) = \frac{2 \sin 5^\circ}{3.0126...}$

$(\theta - 50^\circ) = 3.3169...^\circ$

$\theta = 53.316...^\circ$

The resultant force has a magnitude of 3.01 N (3s.f.) and acts at 53.3° above the horizontal (3s.f.).



4 a Resolving horizontally:

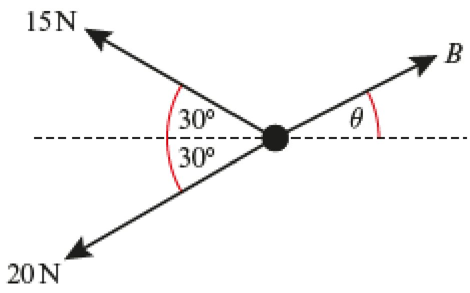
$B\cos\theta = 15\cos 30^\circ + 20\cos 30^\circ$

$B\cos\theta = 35\frac{\sqrt{3}}{2}$ **(1)**

Resolving vertically:

$B\sin\theta = -15\sin 30^\circ + 20\sin 30^\circ$

$B\sin\theta = \frac{5}{2}$ **(2)**



$\mathbf{(2)} \div \mathbf{(1)} \Rightarrow$

$\frac{B\sin\theta}{B\cos\theta} = \frac{5}{2} \times \frac{2}{35\sqrt{3}}$

$\tan\theta = \frac{1}{7\sqrt{3}}$

$\theta = 4.7150...$

$\mathbf{(1)}^2 + \mathbf{(2)}^2 \Rightarrow$

$B^2\left(\cos^2\theta + \sin^2\theta\right) = \left(35\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{5}{2}\right)^2$

$B^2 = \frac{(35^2 \times 3) + 25}{4}$

$B = \frac{\sqrt{3700}}{2}$

$= 30.413...$

B has a magnitude of 30.4 N (3s.f.) and acts at 4.72 to the horizontal (3s.f.).

b Resolving horizontally:

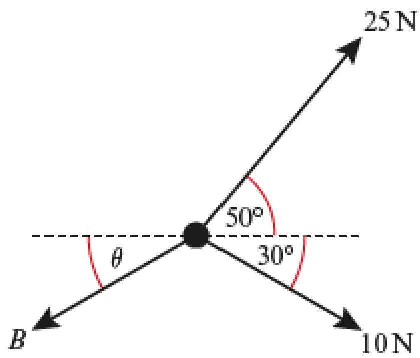
$B\cos\theta = 25\cos 50^\circ + 10\cos 30^\circ$

$B\cos\theta = 24.729...$ **(1)**

Resolving vertically:

$B\sin\theta = -10\sin 30^\circ + 25\sin 50^\circ$

$B\sin\theta = 14.151...$ **(2)**



$\mathbf{(2)} \div \mathbf{(1)} \Rightarrow$

$\frac{B\sin\theta}{B\cos\theta} = \frac{14.151...}{24.729...}$

$\tan\theta = 0.57224...$

$\theta = 29.779...$

$\mathbf{(1)}^2 + \mathbf{(2)}^2 \Rightarrow$

$B^2\left(\cos^2\theta + \sin^2\theta\right) = 14.151...^2 + 24.729...^2$

$B = \sqrt{811.77...}$

$= 28.491...$

B has a magnitude of 28.5 N (3s.f.) and acts at 29.8°below the horizontal (3s.f.).

4 c Resolving horizontally:

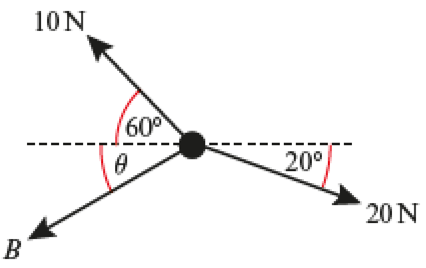
$B\cos\theta = 20\cos 20^\circ - 10\cos 60^\circ$

$B\cos\theta = 13.793...$ **(1)**

Resolving vertically:

$B\sin\theta = 10\sin 60^\circ - 20\sin 20^\circ$

$B\sin\theta = 1.8198...$ **(2)**



$\mathbf{(2)} \div \mathbf{(1)} \Rightarrow$

$\frac{B\sin\theta}{B\cos\theta} = \frac{1.8195...}{13.793...}$

$\tan\theta = 0.13193...$

$\theta = 7.5157...$

$\mathbf{(1)}^2 + \mathbf{(2)}^2 \Rightarrow$

$B^2\left(\cos^2\theta + \sin^2\theta\right) = 1.8195...^2 + 13.793...^2$

$B = \sqrt{193.55...}$

$= 13.912...$

B has a magnitude of 13.9 N (3s.f.) and acts at 7.52° below the horizontal (3s.f.).

5 a Using Newton's second law and resolving horizontally:

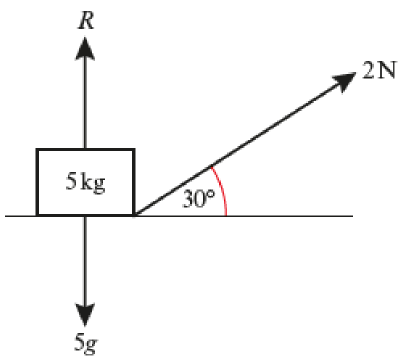
$F = ma$

$2\cos 30^\circ = 5a$

$2\frac{\sqrt{3}}{2} = 5a$

$a = \frac{\sqrt{3}}{5}$

The box accelerates at $\frac{\sqrt{3}}{5}$ ms^{-2}



b Resolving vertically:

$5g = R + 2\sin 30^\circ$

$R = (5 \times 9.8) - 1$

$R = 48$

The normal reaction of the box with the floor is 48 N.

6 Using Newton's second law and resolving horizontally:

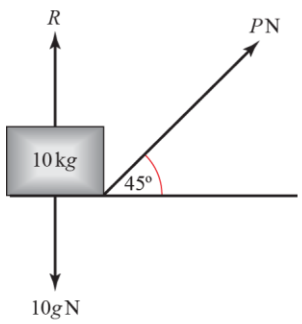
$F = ma$

$P\cos 45^\circ = 10 \times 2$

$P = \frac{20}{\cos 45^\circ}$

$P = 20\sqrt{2}$

The force P is $20\sqrt{2}$ N.



7 Using Newton's second law and resolving horizontally:

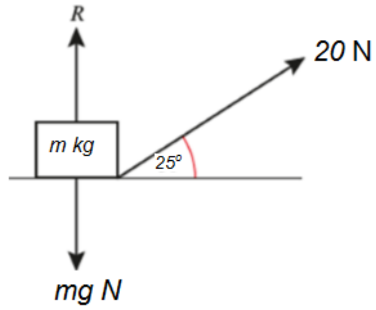
$$F = ma$$

$$20\cos 25^\circ = 0.5m$$

$$m = 2 \times 20\cos 25^\circ$$

$$m = 36.252...$$

The mass of the box is 36.3 kg.



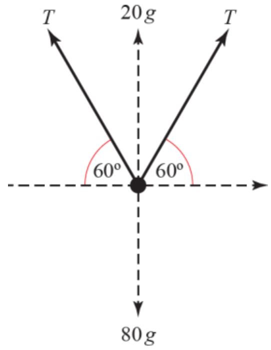
8 Resolving vertically:

$$20g + 2T\sin 60^o = 80g$$

$$2T\sin 60^o = 80g - 20g$$

$$2T\frac{\sqrt{3}}{2} = 60g$$

$$T = \frac{60g}{\sqrt{3}} = 20\sqrt{3}g \quad \text{as required.}$$



9 Resolving vertically:

$$2 = 12 - F_2 \sin 30^\circ$$

$$F_2 = \frac{12-2}{\sin 30^\circ}$$

$$F_2 = 20$$

Resolving horizontally:

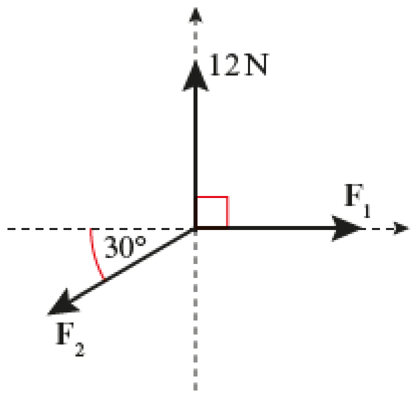
$$2\sqrt{3} = F_1 - F_2\cos 30^\circ$$

$$F_1 = 2\sqrt{3} + 20\cos 30^\circ$$

$$F_1 = 2\sqrt{3} + \frac{20\sqrt{3}}{2}$$

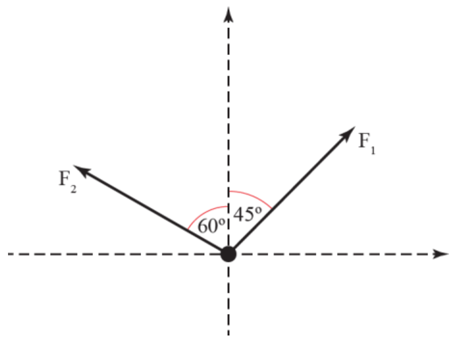
$$F_1 = 12\sqrt{3}$$

The forces F_1 and F_2 are $12\sqrt{3}$ N and 20 N respectively.



Challenge

Resolving vertically:



$$5 = F_1 \cos 45^\circ + F_2 \cos 60^\circ$$

$$5 = \frac{F_1}{\sqrt{2}} + \frac{F_2}{2}$$

$$\frac{F_1}{\sqrt{2}} = 5 - \frac{F_2}{2} \qquad \textbf{(1)}$$

Resolving horizontally:

$$3 = F_1 \sin 45^\circ - F_2 \sin 60^\circ$$

$$3 = \frac{F_1}{\sqrt{2}} - \frac{F_2\sqrt{3}}{2} \qquad \textbf{(2)}$$

Substituting $\frac{F_1}{\sqrt{2}} = 5 - \frac{F_2}{2}$ from (**1**), in (**2**):

$$3 = 5 - \frac{F_2}{2} - \frac{F_2\sqrt{3}}{2}$$

$$2 = \frac{F_2}{2} + \frac{F_2\sqrt{3}}{2}$$

$$4 = (\sqrt{3} + 1)F_2$$

$$F_2 = \frac{4}{\sqrt{3} + 1}$$

$$F_2 = \frac{4(\sqrt{3} - 1)}{3 - 1}$$

$$F_2 = 2\sqrt{3} - 2$$

Substituting $F_2 = 2\sqrt{3} - 2$ in (**1**):

$$\frac{F_1}{\sqrt{2}} = 5 - \left(\frac{2\sqrt{3} - 2}{2}\right)$$

$$\frac{F_1}{\sqrt{2}} = 6 - \sqrt{3}$$

$$F_1 = 6\sqrt{2} - \sqrt{6}$$

The forces F_1 and F_2 are $6\sqrt{2} - \sqrt{6}$ N and $2\sqrt{3} - 2$ N respectively.

