Projectiles Mixed exercise 6

1 a Resolving the initial velocity vertically

$R(\uparrow)\ u_y = 42\sin 45°$

$= 21\sqrt{2}$

$u = 21\sqrt{2},\ v = 0,\ a = -9.8,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = (21\sqrt{2})^2 - 2\times 9.8\times s$

$s = \frac{(21\sqrt{2})^2}{2\times 9.8} = \frac{882}{19.6} = 45$

The greatest height above the plane reached by P is 45 m.

b $R(\uparrow)$

$u = 21\sqrt{2},\ s = 0,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 21\sqrt{2}t - 4.9t^2$

$t \neq 0$

$t = \frac{21\sqrt{2}}{4.9} = 6.0609...$

The time of flight of P is 6.1 s (2 s.f.).

2 Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = 21$

$R(\uparrow)\ u_y = 0$

Resolve horizontally to find the time of flight:

$R(\rightarrow):\ s = 56,\ u = 21,\ t = ?$

$s = ut$

$56 = 21\times t$

$t = \frac{56}{21} = \frac{8}{3}$

Resolve vertically with $t = \frac{8}{3}$ s to find h

$R(\downarrow):\ u = 0,\ s = h,\ a = 9.8,\ t = \frac{8}{3}$

$s = ut + \tfrac{1}{2}at^2$

$h = 0 + 4.9\left(\frac{8}{3}\right)^2 = 34.844$

$h = 35$ (2 s.f.)

3 a $\tan\theta = \frac{4}{3} \Rightarrow \sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$

Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = 15\cos\alpha = 15\times\frac{3}{5} = 9$

$R(\uparrow)\ u_y = 15\sin\alpha = 15\times\frac{4}{5} = 12$

$R(\rightarrow):\ u = 9,\ t = 4,\ s = ?$

$$\begin{aligned} s &= ut \\ &= 9\times 4 \\ &= 36 \end{aligned}$$

The horizontal distance between the point of projection and the point where the ball hits the lawn is 36 m.

b Let the vertical height above the lawn from which the ball was thrown be h m

$R(\uparrow):\ u = 12,\ \ s = -h,\ a = -9.8,\ \ t = 4$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ -h &= 12\times 4 - 4.9\times 4^2 \\ &= -30.4 \\ \Rightarrow h &= 30.4 \end{aligned}$$

The vertical height above the lawn from which the ball was thrown is 30 m (2 s.f.).

4 a Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = 40\cos 30° = 20\sqrt{3}$

$R(\uparrow)\ u_y = 40\sin 30° = 20$

First, resolve vertically to find the time of flight:

$R(\uparrow):\ u = 20,\ \ s = 0,\ \ a = -9.8,\ \ t = ?$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 0 &= 20t - 4.9t^2 \\ 0 &= t(20 - 4.9t) \\ t \neq 0 &\Rightarrow t = \frac{20}{4.9} \end{aligned}$$

Now resolve horizontally with $t = \frac{20}{4.9}$ to find distance AB

$R(\rightarrow):\ u = v = 20\sqrt{3},\ t = \frac{20}{4.9},\ s = ?$

$$\begin{aligned} s &= ut \\ &= 20\sqrt{3}\times\frac{20}{4.9} = 141.39... \end{aligned}$$

$AB = 140$ (2 s.f.)

4 b $R(\uparrow)$: $u = 20,\ v = v_y,\ a = -9.8,\ s = 15$

$$v^2 = u^2 + 2as$$
$$v_y^2 = 20^2 - 2\times 9.8\times 15 = 106$$
$$V^2 = u_x^2 + v_y^2 = \left(20\sqrt{3}\right)^2 + 106 = 1306$$
$$V = \sqrt{1306} = 36.138...$$

The speed of the projectile at the instants when it is 15 m above the plane is $36\,\mathrm{m\,s^{-1}}$ (2 s.f.)

5 a Taking components of velocity horizontally and vertically:

$R(\rightarrow)$ $u_x = U\cos\theta$

$R(\uparrow)$ $u_y = U\sin\theta$

First resolve vertically to find time of flight:

$R(\uparrow)$: $u = U\sin\theta,\ a = -g,\ s = 0,\ t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$0 = (U\sin\theta)\times t - \frac{1}{2}gt^2$$
$$0 = t\left(U\sin\theta - \frac{1}{2}gt\right)$$
$$t = \frac{2u\sin\theta}{g} \quad \text{(since } t = 0 \text{ corresponds to launch)}$$

Let the range be R. Resolve horizontally with $t = \dfrac{2u\sin\theta}{g}$ to find R:

$R(\rightarrow)$: $u = U\cos\theta,\ s = R,\ t = \dfrac{2u\sin\theta}{g}$

$$s = vt$$
$$R = U\cos\theta\times\frac{2U\sin\theta}{g}$$
$$= \frac{2U\sin\theta\cos\theta}{g}$$

Using the identity $\sin 2\theta = 2\sin\theta\cos\theta$

$$R = \frac{U^2\sin 2\theta}{g}$$

b R is a maximum when $\sin 2\theta = 1$, that is when $\theta = 45°$

The maximum range of the projectile is $\dfrac{U^2}{g}$

c $R = \dfrac{U^2\sin 2\theta}{g} = \dfrac{2U^2}{3g}$

$$\Rightarrow \sin 2\theta = \frac{2}{3}$$
$$2\theta = 41.81°,\ (180 - 41.81)°$$
$$\theta = 20.9°,\ 69.1°,\ \text{(nearest } 0.1°\text{)}$$

6 Taking components horizontally and vertically

$R(\rightarrow)\ \ u_x = 40\cos 30° = 20\sqrt{3}$

$R(\uparrow)\ \ u_y = 40\sin 30° = 20$

a $R(\uparrow)$: $u = 20,\ v = 0,\ a = -g,\ t = ?$

$v = u + at$

$0 = 20 - 9.8t$

$t = \dfrac{20}{9.8} = 2.0408...$

The time taken by the ball to reach its greatest height above A is 2.0 s (2 s.f.)

b Resolve vertically with $s = 15.1$ m to find time of flight.

$R(\uparrow)$: $u = 20,\ s = 15.1,\ a = -g,\ t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$15.1 = 20t - 4.9t^2$$

$4.9t^2 - 20t + 15.1 = 0$

$(t-1)(4.9t - 15.1) = 0$

On the way down the time must be greater than the result in part **a**, so $t \neq 1$

$\Rightarrow t = \dfrac{15.1}{4.9} = 3.0816...$

The time taken for the ball to travel from A to B is 3.1s (2 s.f.)

c $R(\uparrow)$: $u = 20,\ a = -g,\ t = \dfrac{15.1}{4.9},\ v = v_y$

$v_y = u + at$

$v_y = 20 - 9.8 \times \dfrac{15.1}{4.9}$

$= -10.2$

$R(\rightarrow)\ \ v_x = u_x = 20\sqrt{3}$

Hence:

$V^2 = u_x^2 + v_y^2$

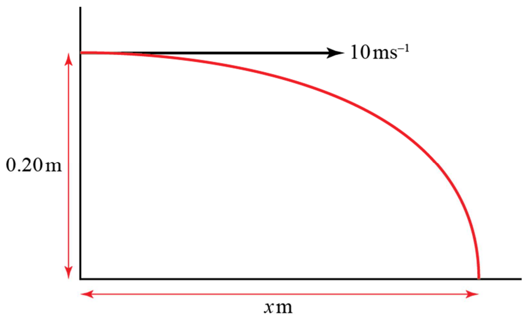
$= \left(20\sqrt{3}\right)^2 + (-10.2)^2$

$= 1304.04$

$V = \sqrt{1304.04} = 36.111...$

The speed with which the ball hits the hoarding is $36\,\mathrm{m\,s^{-1}}$ (2 s.f.).

7 a Let downwards be the positive direction.



First, resolve vertically to find the time of flight:
R(↓): $u = u_y = 0$, $a = g = 10$ ms^{-2}, $s = 20$ cm $= 0.20$ m, $t = ?$

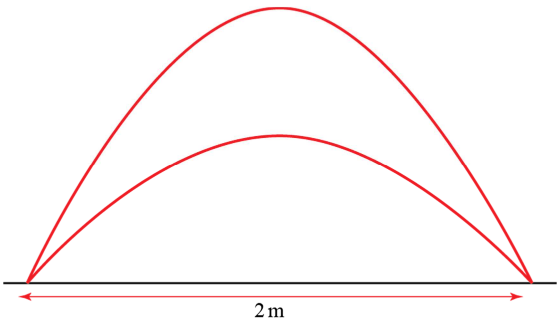
$$s = ut + \frac{1}{2}at^2$$
$$0.2 = 0 + \frac{1}{2} \times 10 \times t^2$$
$$t^2 = \frac{0.2}{5}$$
$$t = 0.2$$

Let the horizontal distance to the target be x m.
R(→): $v = u_x = 10$ ms^{-1}, $t = 0.2$ s, $s = x$

$$s = vt$$
$$x = 10 \times 0.2$$
$$x = 2$$

The target is 2 m from the point where the ball was thrown.

b Using the equation



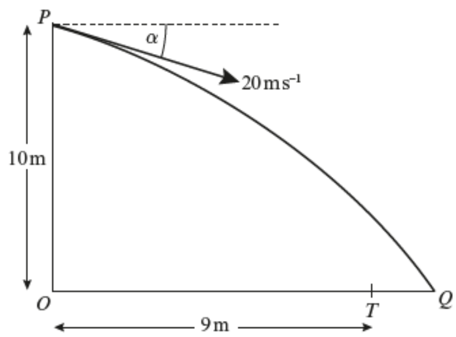
$$\text{Range} = \frac{U^2 \sin 2\alpha}{10}$$

gives:

$$2 = 10 \sin 2\alpha$$
$$\sin 2\alpha = 0.2$$
$$2\alpha = 11.536... \Rightarrow \alpha = 5.7684...$$
or
$$2\alpha = 168.46... \Rightarrow \alpha = 84.231...$$

For the ball to pass through the hole the boy must throw the ball at 5.77° or 84.2° above the horizontal (both angles to 3s.f.).

8 Let downwards be the positive direction.



$\tan \alpha = \frac{3}{4}$ so $\sin \alpha = \frac{3}{5}$ and $\cos \alpha = \frac{4}{5}$

a R(↓): $u_y = 20 \sin \alpha = 12$ ms^{-1}, $a = g = 10$ ms^{-2}, $s = 10$ m, t = ?

$$s = ut + \frac{1}{2}at^2$$
$$10 = 12t + \frac{1}{2}10t^2$$
$$0 = 5t^2 + 12t - 10$$
$$t = \frac{-12 \pm \sqrt{144 - (4 \times 5 \times (-10))}}{10}$$
$$t = 0.65472... \text{ or } -3.0547$$

The negative answer does not apply, so the time taken to travel PQ is 0.65 s (2s.f.).

8 b First, find OQ:

R(→): $v = u_x = 20\cos\alpha = 16$, $s = 10$, $t = 0.65472\ldots$

$s = vt$

$OQ = 16 \times 0.65472...$

$= 10.475...$

Next find TQ:

$TQ = OQ - 9$

$= 10.475... - 9$

$= 1.475...$

The distance TQ is 1.5 m (2s.f.).

c First, resolve horizontally to find the time at which the ball passes through A

R(→): $v_x = u_x = 20\cos\alpha = 16$, $s = 9$, $t = ?$

$s = vt$

$9 = 16 \times t$

$t = 0.5625$

Then resolve vertically with $t = 0.5625$ to find vertical speed of ball as it passes through A

R(↓): $u_y = 20\sin\alpha = 12$, $a = g = 10$, $v_y = ?$

$v = u + at$

$v_y = 12 + (10 \times 0.5625)$

$v_y = 17.625$

The speed of ball at A is given by:

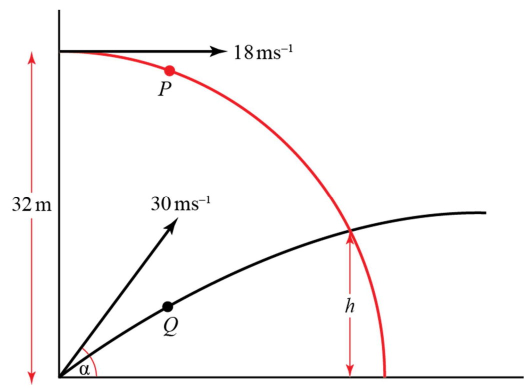
$v^2 = v_x^2 + v_y^2$

$v^2 = 16^2 + 17.625^2$

$v = \sqrt{566.64...} = 23.804...$

The speed of the ball at A is 23.8 ms^{-1} (3s.f.).

9 Let u_{P_x} denote the horizontal component of the initial velocity of P, and u_{Q_y} denote the vertical component of the initial velocity of Q, etc.



a For P: R(→): $v = u_{P_x} = 18$

$v = u_{Q_x} = 30\cos\alpha$

Since the balls eventually collide, these two speeds must be the same, so:

$30\cos\alpha = 18$

$\cos\alpha = \frac{18}{30} = \frac{3}{5}$ as required.

9 b Since $\cos\alpha = \frac{3}{5} \Rightarrow \sin\alpha = \frac{4}{5}$

Suppose the balls collide at a height h above the ground.

Resolve the vertical motion of both P and Q to find two equations for h in terms of t. We can then equate the two to solve for t.

For P, R(↓): $u = u_{P_y} = 0$, $a = g$, $s = 32 - h$, $t = t$

$$s = ut + \frac{1}{2}at^2$$

$$32 - h = 0 + \frac{1}{2}gt^2$$

$$h = 32 - \frac{1}{2}gt^2 \qquad \textbf{(1)}$$

For Q, R(↑): $u = u_{Q_y} = 30\sin\alpha = 24$, $a = -g$, $s = h$, $t = t$

$$s = ut + \frac{1}{2}at^2$$

$$h = 24t - \frac{1}{2}gt^2 \qquad \textbf{(2)}$$

(1) = **(2)**:

$$32 - \frac{1}{2}gt^2 = 24t - \frac{1}{2}gt^2$$

$$24t = 32$$

$$t = \frac{32}{24} = \frac{4}{3}$$

The balls collide after $\frac{4}{3}$ s of flight.

Challenge

The vertical motion of the golf ball is unaffected by the motion of the ship and, therefore, the time of flight is given by the usual equation for the time of flight of a projectile:

$$T = \frac{2v\sin\alpha}{g} = \frac{2v\sin 60^\circ}{g}$$

The absolute path of the ball is a parabola, and the horizontal component of the velocity is, as usual, constant.

However, the ball's horizontal speed relative to the ship is not constant: the ball appears to decelerate at the same rate as the ship is accelerating and the path appears to be non-symmetrical.

Therefore, considering the horizontal motion of the ball:

R(→): $s = 250$ m, a $= -1.5$ ms^{-2}, $t = T = \dfrac{2v\sin 60^\circ}{g}$ s, $u = v_x = v\cos 60^\circ$ ms^{-1}

$$s = ut + \frac{1}{2}at^2$$

$$250 = v\cos 60^\circ\left(\frac{2v\sin 60^\circ}{g}\right) - \frac{1.5}{2}\left(\frac{2v\sin 60^\circ}{g}\right)^2$$

$$250 = \frac{v^2 \times 2\cos 60^\circ \sin 60^\circ}{g} - \frac{3v^2 \times \sin^2 60^\circ}{g^2}$$

$$250g^2 = \left(g\sin 120^\circ - 3\sin^2 60^\circ\right)v^2$$

$$v^2 = \frac{250 \times 9.8^2}{\left(\frac{\sqrt{3}}{2} \times 9.8\right) - \left(3 \times \frac{3}{4}\right)}$$

$$v = \sqrt{3849.5...} = 62.044...$$

The initial speed of the golf ball is 62 ms^{-1} (to 2s.f.).

[Note that the equation above can be written:

$$250 + \frac{3}{4}\left(\frac{2v\sin 60^\circ}{g}\right)^2 = \frac{v^2\sin 120^\circ}{g}$$

The additional term on the LHS is the distance covered by the ship during the time of flight of the ball, and the RHS is the usual equation for the range of a projectile.]