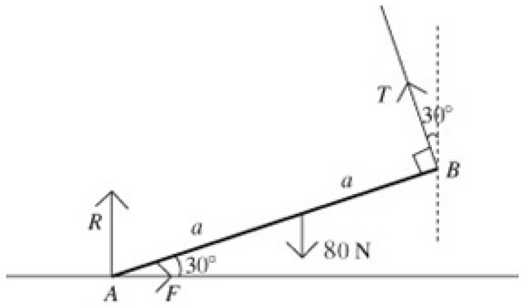
Applications of forces 7D

1 a Suppose that the rod has length $2a$.



Taking moments about A:

$2aT = 80 \times a\cos 30°$

$2T = 80 \times \frac{\sqrt{3}}{2}$

$T = 20\sqrt{3}$

$= 34.6 \text{ N}$

$R(\rightarrow)$, $\quad F = T\sin 30° = 10\sqrt{3} = 17.3 \text{ N}$

$R(\uparrow)$, $\quad T\cos 30° + R = 80$

$R = 80 - 20\sqrt{3} \times \frac{\sqrt{3}}{2}$

$= 50 \text{ N}$

In order for the rod to remain in equilibrium, we must have $F \leqslant \mu R$:

$10\sqrt{3} \leqslant \mu \times 50$

$\mu \geqslant \frac{10\sqrt{3}}{50}$

$\mu \geqslant \frac{\sqrt{3}}{5}$

$\therefore$ minimum $\mu = 0.35$ (2 s.f.)

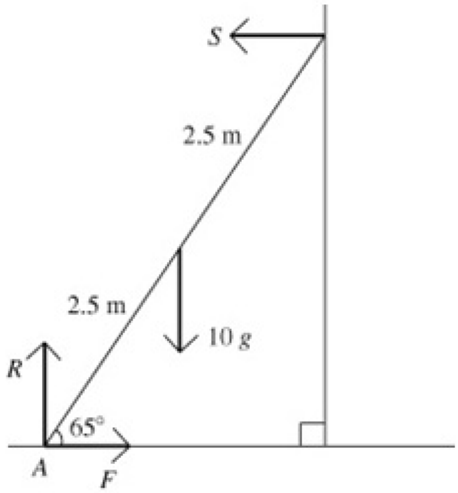
So $T = 34.6$ N, $F = 17.3$ N, $R = 50$ N, minimum $\mu = 0.35$

b Reaction at floor will be resultant of R and F

Magnitude = $\sqrt{50^2 + 17.3^2} = 53$ N (2 s.f.)

Angle above horizontal = $\tan^{-1}\left(\frac{50}{17.3}\right) = 71°$ (2 s.f.)

2 Let A be the end of the ladder on the ground.
Let F be the frictional force at A.



a Taking moments about A:
$10g \times 2.5\cos 65° = S \times 5\sin 65°$

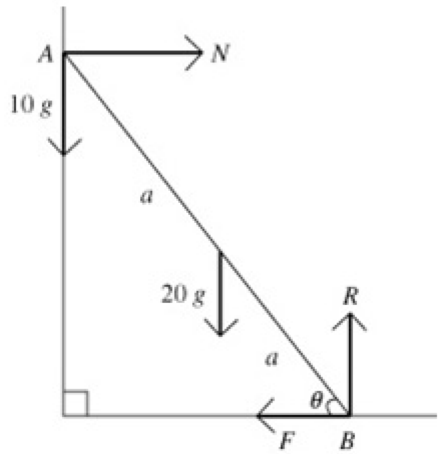
$$S = \frac{25g\cos 65°}{5\sin 65°} = \frac{5g}{\tan 65°} = 22.8\text{ N}$$

b $R(\rightarrow)$, $F = S = 22.8\text{ N}$
$R(\uparrow)$, $R = 10g = 98\text{ N}$

c To ensure ladder remains in equilibrium, we must have
$F \leqslant \mu R$
$22.8 \leqslant \mu \times 98$
$\mu \geqslant 0.233$ (3 s.f.)

d The weight is shown as acting through the midpoint of the ladder because of the assumption that the ladder is uniform.

3 Let the ladder have length $2a$, and be inclined at θ to the horizontal.



a $R(\uparrow)$, $R = 30g$
Taking moments about A:
$20g \times a\cos\theta + F \times 2a\sin\theta = R \times 2a\cos\theta$

$$20g\cos\theta + 2F\sin\theta = 60g\cos\theta \quad (\text{using } R = 30g)$$
$$2F\sin\theta = 40g\cos\theta$$
$$F = \frac{20g}{\tan\theta}$$

The ladder is on the point of slipping, so $F = \mu R$

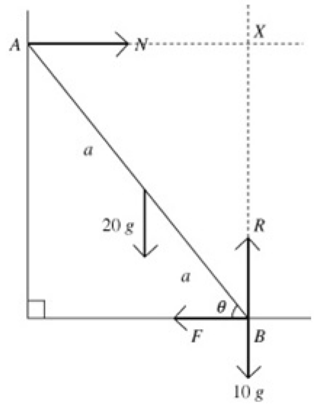
$$\frac{20g}{\tan\theta} = \frac{3}{4} \times 30g$$
$$\therefore \tan\theta = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$
$$\therefore \theta = 41.6°$$

N is the normal reaction at A,
R is the normal reaction at B,
F is the frictional force at B.

3 b $R(\uparrow), \quad R = 30g$

$R(\rightarrow), \quad N - F = 0$

$N = F$



Taking moments about B:

$20g \times a\cos\theta = N \times 2a\sin\theta$

$20g \times a\cos\theta = F \times 2a\sin\theta$

$F = \dfrac{10g\cos\theta}{\sin\theta}$

$F = \dfrac{10g}{\tan\theta}$

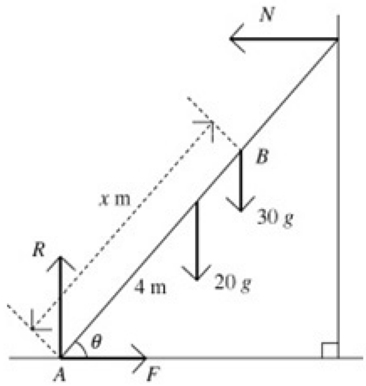
The ladder is on the point of slipping, so $F = \mu R$

$\dfrac{10g}{\tan\theta} = \dfrac{3}{4} \times 30g$

$\tan\theta = \dfrac{4}{9}$

$\theta = 24.0°$

c The assumption that the wall is smooth means there is no friction between the ladder and the wall.



4 a Suppose that the boy reaches the point B, a distance x from A, whilst the end of the ladder is still in contact with the ground.

$R(\rightarrow)$, $F = N$

$R(\uparrow)$, $R = 50g$

Taking moments about A:

$$20g \times 4\cos\theta + 30g \times x\cos\theta = N \times 8\sin\theta$$

$$80g + 30gx = 8N\tan\theta$$

$$N = \frac{80g + 30gx}{8\tan\theta}$$

$$N = \frac{80g + 30gx}{16} \quad \text{(since } \tan\theta = 2\text{)}$$

$$F = \frac{80g + 30gx}{16} \quad \text{(since } F = N\text{)}$$

$$\mu R = \frac{80g + 30gx}{16} \quad \text{(in limiting equilibrium)}$$

$$0.3 \times 50g = \frac{80g + 30gx}{16}$$

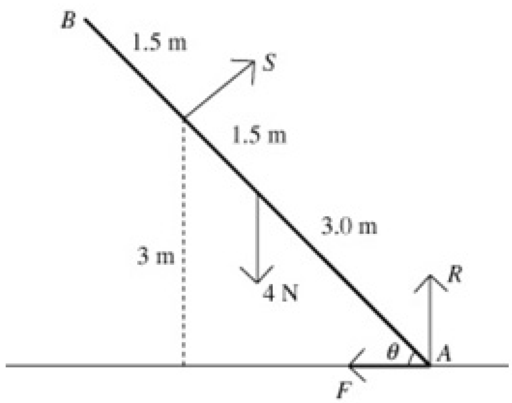
$$240 = 80 + 30x$$

$$x = 5\frac{1}{3} \text{ m}$$

b i The ladder may not be uniform.

ii There would be friction between the ladder and the wall.

5 Let:
S be the normal reaction of the rail on the pole at C,
R be the normal reaction of the ground on the pole at A,
F be the friction between the pole and the ground at A.
θ be the angle between the pole and the ground.



From the diagram,

$$\sin\theta = \frac{3}{4.5} = \frac{2}{3} \text{ and hence } \cos\theta = \frac{\sqrt{9-4}}{3} = \frac{\sqrt{5}}{3}$$

a Taking moments about A:

$$4.5S = 4 \times 3\cos\theta$$

$$= \frac{12\sqrt{5}}{3}$$

$$= 4\sqrt{5}$$

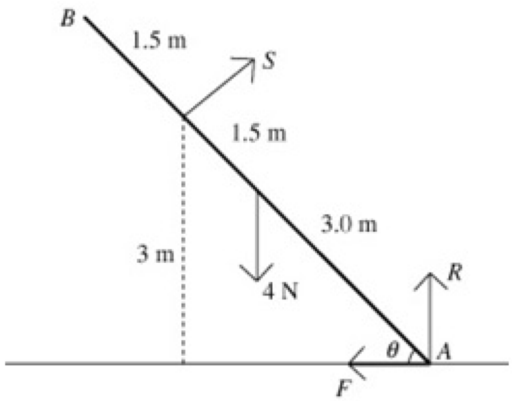
$$S = \frac{8\sqrt{5}}{9} \text{ N}$$

5 b $R(\rightarrow)$

$F = S\sin\theta$

$= \frac{8\sqrt{5}}{9} \times \frac{2}{3}$

$= \frac{16\sqrt{5}}{27}$



$R(\uparrow)$

$R + S\cos\theta = 4$

$R = 4 - \frac{8\sqrt{5}}{9} \times \frac{\sqrt{5}}{3}$

$= 4 - \frac{40}{27}$

$= \frac{68}{27}$

Pole is in limiting equilibrium, so $F = \mu R$

$\frac{16\sqrt{5}}{27} = \mu \times \frac{68}{27}$

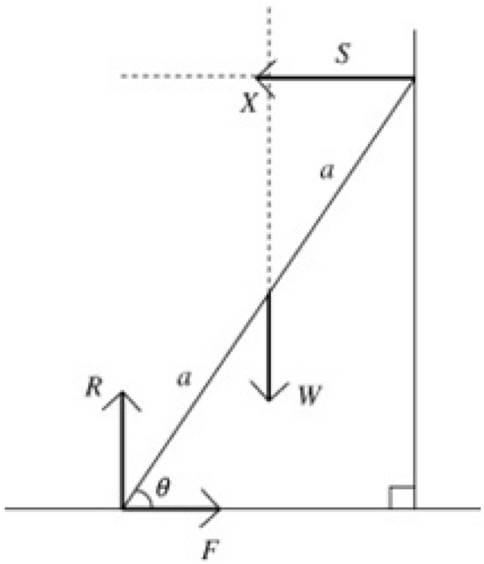
$\therefore \mu = \frac{16\sqrt{5}}{68}$

$= \frac{4\sqrt{5}}{17}$

$= 0.526$ (3 s.f.)

c The assumption that the rail is smooth means there is no friction between the rail and the pole.

6 Suppose that the ladder has length $2a$ and weight W.
Let:
S be the normal reaction of the wall on the ladder,
R be the normal reaction of the floor on the ladder,
F be the friction between the floor and the ladder.
X be the point where the lines of action of W and S meet.



Taking moments about X:

$2a\sin\theta \times F = R \times a\cos\theta$

$2F\sin\theta = R\cos\theta$ **(1)**

The ladder is in limiting equilibrium, so $F = \mu R$

Substituting $F = \mu R$ in (**1**):

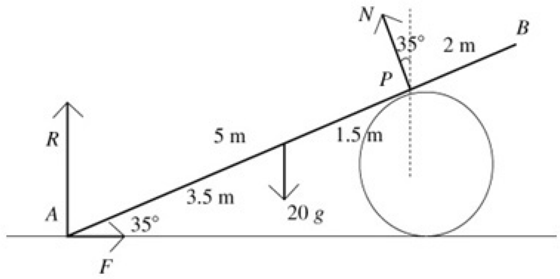
$2\mu R\sin\theta = R\cos\theta$

$2\mu\sin\theta = \cos\theta$

$\frac{2\mu\sin\theta}{\cos\theta} = 1$

$2\mu\tan\theta = 1$

7 Let:
N be the normal reaction of the drum on the ladder at P,
R be the normal reaction of the ground on the ladder at A,
F be the friction between the ground and the ladder at A.



Taking moments about A:
$20g\times 3.5\cos 35° = 5N$

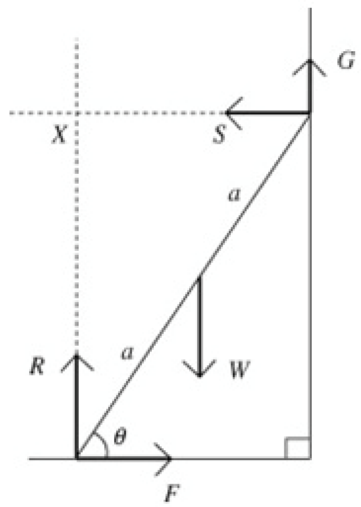
$$N = \frac{20g\times 3.5\cos 35°}{5} = 14g\cos 35°$$

$R(\uparrow)$
$N\cos 35° + R = 20g$
$$R = 20g - 14g\cos 35°\times\cos 35° = 103.9...\text{ N}$$

$R(\rightarrow)$
$$F = N\sin 35° = 14g\cos 35°\times\sin 35° = 64.46...\text{ N}$$

$F \leqslant \mu R$ to maintain equilibrium:
$14g\cos 35°\sin 35° \le \mu(20g - 14g\cos^2 35°)$
$$\mu \ge \frac{14\cos 35°\sin 35°}{20 - 14\cos^2 35°}$$
$\mu \ge 0.620$ (3 s.f.)
Least possible μ is 0.620 (3 s.f.)

8 Let:
R be the reaction of the ground on the ladder
F be the friction between the ground and the ladder
S be the reaction of the wall on the ladder
G be the friction between the wall and the ladder.
X be the point where the lines of action R and S meet.



Suppose that the ladder has length $2a$ and weight W.

As the ladder rests in limiting equilibrium, $F = \mu_1 R$ and $G = \mu_2 S$.

Taking moments about X:
$W\times a\cos\theta = F\times 2a\sin\theta + G\times 2a\cos\theta$
$$W = 2F\tan\theta + 2G \quad \textbf{(1)}$$

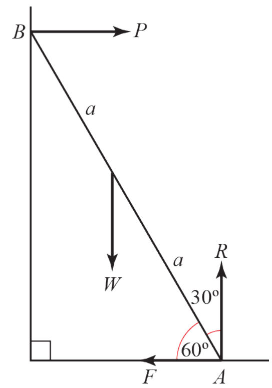
$R(\rightarrow)$, $F = S$
$R(\uparrow)$, $W = R + G$

8 Substituting for W and F in equation (**1**):

$$R+G=2\mu_1 R\tan\theta+2G$$
$$R-G=2\mu_1 R\tan\theta$$
$$R-\mu_1\mu_2 R=2\mu_1 R\tan\theta \qquad (\text{Since } G=\mu_2 S=\mu_2 F=\mu_2\mu_1 R)$$

Hence $\dfrac{1-\mu_1\mu_2}{2\mu_1}=\tan\theta$

9 Let:
A and B be the ends of the ladder.
P be the normal reaction of the wall on the ladder at B,
R the normal reaction of the ground on the ladder at A
F be the friction at between the ladder and the ground at A



Let the length of the ladder be $2a$.

a Taking moments about A:
$W\times a\cos 60°=P\times 2a\cos 30°$

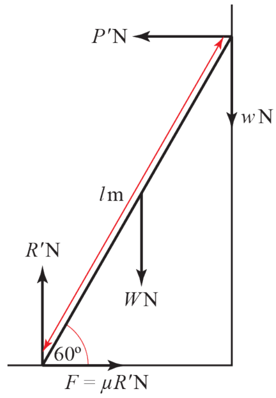
$$P=\frac{Wa\cos 60°}{2a\cos 30°}$$
$$P=\frac{W\times\frac{1}{2}}{2\times\frac{\sqrt{3}}{2}}$$
$$P=\frac{W}{2\sqrt{3}} \qquad (\mathbf{1})$$

b $R(\uparrow)$, $\quad R=W \qquad$ (**2**)
$R(\rightarrow)$, $\quad F=P \qquad$ (**3**)

Now $F\leqslant\mu R$ since the ladder is in equilibrium (if not, ladder would slide)
Hence, $\quad P\leqslant\mu R \quad$ (by (**3**))

$$\frac{W}{2\sqrt{3}}\leqslant\mu R \quad \text{(by (1))}$$
$$\frac{W}{2\sqrt{3}}\leqslant\mu W \quad \text{(by (2))}$$
$$\mu\geqslant\frac{\sqrt{3}}{6}$$

9 c Let:
R' be the normal reaction of the ground on the ladder at A
P' be the normal reaction of the wall on the ladder at B,
l be the length of the ladder



Since the ladder is in limiting equilibrium, $F' = \mu R'$

$R(\uparrow)$, $R' = W + w$

$R(\rightarrow)$, $\mu R' = P'$

Taking moments about B:

$$\frac{Wl\cos 60^\circ}{2} + \left(F' \times l\sin 60^\circ\right) = \left(R' \times l\cos 60^\circ\right)$$

$$\frac{W}{4} + \left(\mu R' \times \frac{\sqrt{3}}{2}\right) = \frac{R'}{2}$$

$$\frac{W}{4} + \left(\frac{\sqrt{3}}{5}(W+w) \times \frac{\sqrt{3}}{2}\right) = \frac{W+w}{2} \qquad \text{(since } R' = W + w \text{ and } \mu = \frac{\sqrt{3}}{5}\text{)}$$

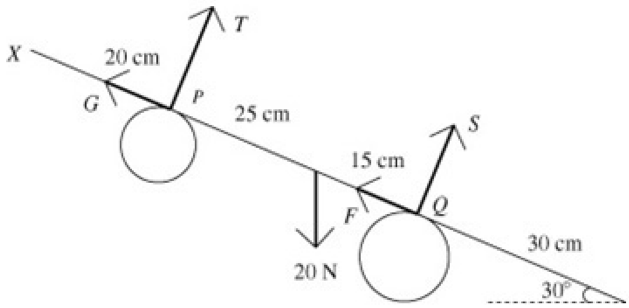
$$W + \frac{6}{5}(W+w) = 2(W+w)$$

$$5W + 6W + 6w = 10W + 10w$$

$$W = 4w$$

$$\Rightarrow w = \frac{W}{4}$$

10 Let:
T be the normal force of the peg on the rod at P,
G be the frictional force at P,
S be the normal force of the peg on the rod at Q,
F be the frictional force at Q.



a Taking moments about P:

$$S \times 40 = 20 \times 25 \times \cos 30°$$

$$S = \frac{20 \times 25 \times \frac{\sqrt{3}}{2}}{40}$$

$$S = \frac{25\sqrt{3}}{4}\text{ N}$$

Taking moments about Q:

$$T \times 40 = 20 \times 15 \times \cos 30°$$

$$T = \frac{20 \times 15 \times \frac{\sqrt{3}}{2}}{40}$$

$$T = \frac{15\sqrt{3}}{4}\text{ N}$$

10 b $R(\searrow)$

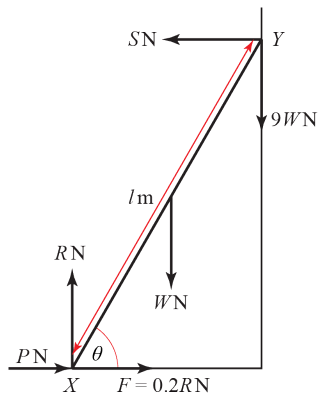
$G + F = 20\cos 60° = 10$ **(1)**

Since the rod is about to slip, friction is limiting and hence $G = \mu T$, $F = \mu S$.
From part **a,**

$G + F = \mu T + \mu S = \mu \times \frac{40\sqrt{3}}{4} = 10\sqrt{3}\mu$ **(2)**

$(\mathbf{1}) = (\mathbf{2}) \Rightarrow \mu = \frac{1}{\sqrt{3}}$

11 a Let:
S be the normal reaction of the wall on the ladder at Y,
R be the normal reaction of the ground on the ladder at X
F be the friction at between the ladder and the ground at X



$\tan\theta = \sqrt{3}$ so $\sin\theta = \frac{\sqrt{3}}{3}$ and $\cos\theta = \frac{1}{2}$

Ladder is in equilibrium.
Taking moments about X:

$\frac{Wl\cos\theta}{2} + 9Wl\cos\theta = Sl\sin\theta$

$\frac{W}{4} + \frac{9W}{2} = \frac{\sqrt{3}S}{2}$

$\sqrt{3}S = \frac{W}{2} + 9W$

$\sqrt{3}S = \frac{19W}{2}$

$S = \frac{19W}{2\sqrt{3}}$

b R(↑): $R = W + 9W = 10W$
For the ladder to be in limiting equilibrium,

$F = \mu R$

$F = \frac{1}{5} \times 10W$

$F = 2W$

R(→):
If $P + F > S$, ladder will slide towards and up the wall
If $P < S - F$, ladder will slide away from and down the wall
Therefore $S - F \le P \le S + F$

Substituting values for S & F from part **a** and above:

$\frac{19W}{2\sqrt{3}} - 2W \le P \le \frac{19W}{2\sqrt{3}} + 2W$

$\left(\frac{19}{2\sqrt{3}} - 2\right)W \le P \le \left(\frac{19}{2\sqrt{3}} + 2\right)W$

11 c Modelling the ladder as uniform allows us to assume the weight acts through the midpoint.

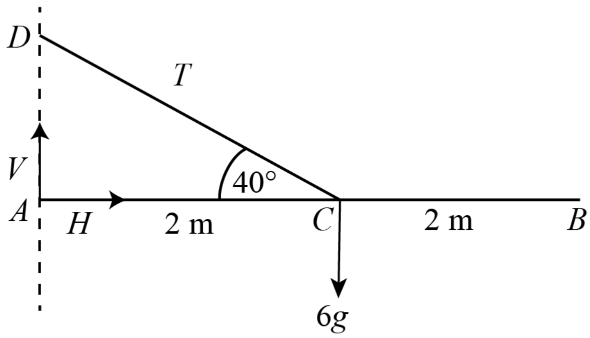
d i The reaction of the wall on the ladder will decrease. To understand why, consider how we took moments about X in part **a**

$$\frac{Wl\cos\theta}{2}+9Wl\cos\theta=Sl\sin\theta$$

The first term in this equation is the turning moment of the weight of the ladder, which acts at a distance $\frac{l}{2}$ from X. If the centre of mass of the ladder is more towards X, say $\frac{l}{a}$ where $a>2$, then this first term would decrease and hence S would also decrease.

ii Ladder remains in equilibrium when $S-F\leq P\leq S+F$
If S were to decrease, then this range of values for P would also decrease.

12 a



Taking moments about A

$M(A): 6g\times 2=2T\sin 40°$

$$T=\frac{6g}{\sin 40°}=91.47656...=91.5\text{ N (1 d.p.)}$$

b Consider all forces acting on AB

$R(\uparrow)$: $V+T\sin 40°=6g$

$V=6\times 9.8-94.47656...\times\sin(40°)=0\text{ N}$

$R(\rightarrow)$: $H=T\cos 40°=70.075...=70.1\text{ N (1 d.p.)}$

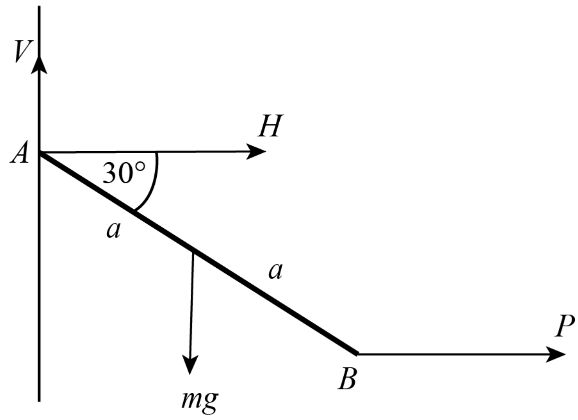
The force exerted on the rod by the wall is 70.1 N parallel to and towards the rod.

13 a Taking moments about A

M(A):

$mg \times a\cos 30° = P \times 2a \times \sin 30°$

$$P = \frac{mga\cos 30°}{2a\sin 30°} = \frac{mga\frac{\sqrt{3}}{2}}{2a\frac{1}{2}} = \frac{\sqrt{3}}{2}mg$$



b $R(\uparrow)$: $V = mg$

R(→): $H = -\frac{\sqrt{3}}{2}mg$

Magnitude of force at the hinge $= \sqrt{(mg)^2 + \left(-\frac{\sqrt{3}}{2}mg\right)^2} = mg\sqrt{1+\frac{3}{4}} = \frac{\sqrt{7}}{2}mg$

Angle of force at hinge $\theta = \arctan\frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 49.1°$ above the horizontal away from the rod.