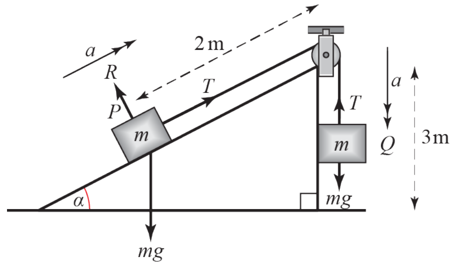
Applications of forces, 7F

1 For P: $R(\nearrow)$

$T - mg\sin\alpha = ma$

$T - \dfrac{3mg}{5} = ma \quad (\mathbf{1})$



For Q: $R(\downarrow)$

$mg - T = ma \quad (\mathbf{2})$

$(\mathbf{1})+(\mathbf{2}): mg - \dfrac{3mg}{5} = 2ma$

$\dfrac{g}{5} = a$

For P:

$u = 0,\ a = \dfrac{g}{5},\ s = 2,\ v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + \dfrac{2g}{5} \times 2$

$v = \sqrt{\dfrac{4g}{5}} = 2.8\,\mathrm{m\,s^{-1}}$

P hits the pulley with speed $2.8\,\mathrm{m\,s^{-1}}$.

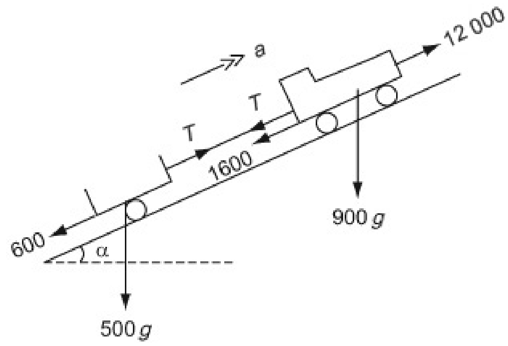
2 $R(\nearrow)$ For the Van:

$F = ma$

$12\,000 - T - 1600 - 900g\sin\alpha = 900a$

$10400 - 900 \times 9.8 \times \dfrac{3}{5} - T = 900a$

$5108 - T = 900a \quad (\mathbf{1})$



$R(\nearrow)$ For the Trailer:

$F = ma$

$T - 600 - 500g\sin\alpha = 500a$

$T - 600 - 500 \times 9.8 \times \dfrac{3}{5} = 500a$

$T - 3540 = 500a \quad (\mathbf{2})$

a $(\mathbf{1})+(\mathbf{2}) \Rightarrow 1568 = 1400a$

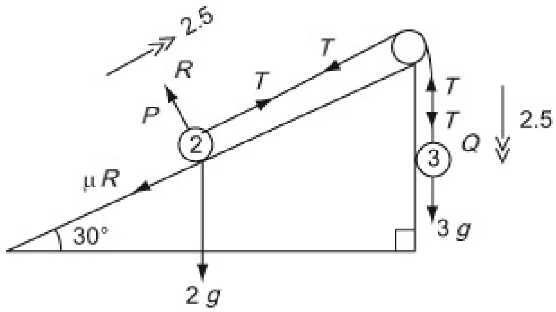
$a = 1.12\ \mathrm{ms^{-2}}$

2 b Sub $a = \frac{1568}{1400}$ ms^{-2} in (**2**)

$$T = 3540 + 500 \times \frac{1568}{1400}$$
$$= 4100 \text{ N}$$

c The resistance forces are unlikely to be constant: it is more probable that they will increase as the speed increases.

3 a For P:



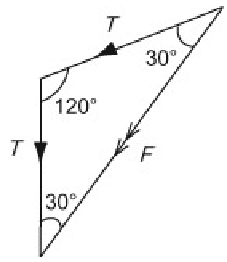
$R(\nwarrow)$
$$R - 2g\cos 30° = 0$$
$$R = g\sqrt{3}$$
$R(\nearrow)$
$$F = ma$$
$$T - \mu R - 2g\cos 60° = 2 \times 2.5$$
$$T - \mu g\sqrt{3} - g = 5 \qquad \textbf{(1)}$$

For Q:

$R(\downarrow)$
$$F = ma$$
$$3g - T = 3 \times 2.5$$
$$3g - T = 7.5 \qquad \textbf{(2)}$$
$$\therefore T = 21.9$$
The tension is 21.9 N.

b $(\textbf{1}) + (\textbf{2}) \Rightarrow \; 2g - \mu g\sqrt{3} = 12.5$
$$\mu g\sqrt{3} = 7.1$$
$$\mu = \frac{7.1}{g\sqrt{3}}$$
$$= 0.418 \text{ (3 s.f.)}$$
The coefficient of friction is 0.42 (2 s.f.).

c $F = 2T\cos 30°$
$$= 43.8\cos 30°$$
$$= 37.9 \text{ N (3 s.f.)}$$



The force exerted by the string on the pulley is 38N (2 s.f.).

4 a For B:

$R(\downarrow)$

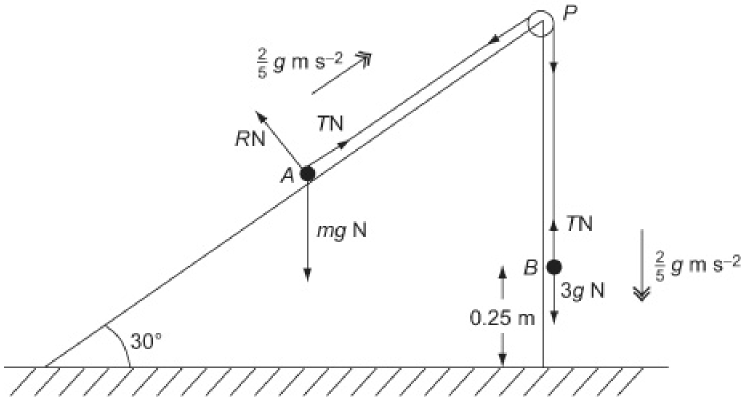
$$F = ma$$

$$3g - T = 3 \times \frac{2}{5}g$$

$$T = 3g - \frac{6}{5}g$$

$$= \frac{9}{5}g$$

$$= 17.64$$



The tension in the string while B is descending is 18 N (2 s.f.).

b For A:

$R(\nearrow)$

$$F = ma$$

$$T - mg\sin 30° = m \times \frac{2}{5}g$$

$$\frac{9}{5}g - \frac{1}{2}mg = \frac{2}{5}mg$$

$$\left(\frac{1}{2} + \frac{2}{5}\right)m = \frac{9}{5}$$

$$\frac{9}{10}m = \frac{9}{5}$$

$$\Rightarrow m = 2$$

4 c Whilst A is still ascending,

$$u = 0,\ a = \frac{2}{5}g,\ s = 0.25,\ v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = \frac{4}{5}g \times 0.25$$

$$v = 1.4\ \text{ms}^{-1}$$

After B strikes the ground, there is no tension in the string and the only force acting on A parallel to the plane is the component of its weight acting down the plane.

For A:

$R(\nearrow)$

$$-mg\sin 30° = ma$$

$$a = -\frac{1}{2}g$$

$$u = 1.4,\ \ v = 0,\ \ a = -\frac{1}{2}g,\ \ t = ?$$

$$v = u + at$$

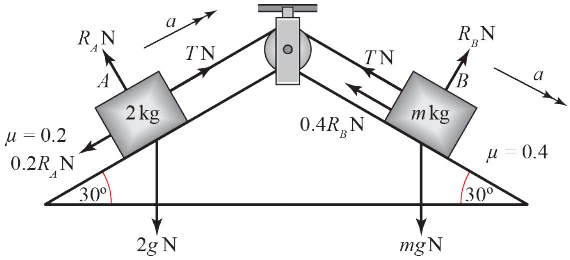
$$0 = 1.4 - \frac{1}{2}gt$$

$$\Rightarrow t = \frac{2.8}{9.8} = \frac{2}{7}$$

The approximate answer, 0.28 s, would also be acceptable.

The time between the instants is $\frac{2}{7}$ s

5 a Let the reaction forces on the blocks be R_A and R_B.
If the system is in limiting equilibrium for the maximum value of m, object B will move down the right-hand slope and object A will move up the left-hand slope.



For A:
$R(\nwarrow)$:
$$R_A = 2g\cos 30^\circ = \sqrt{3}g$$
$R(\nearrow)$
$$T - 2g\sin 30^\circ - 0.2R_A = 0$$
$$T = g + \frac{\sqrt{3}}{5}g$$
$$T = \left(1 + \frac{\sqrt{3}}{5}\right)g \qquad \textbf{(1)}$$

For B
$R(\nearrow)$:
$$R_B = mg\cos 30^\circ = \frac{\sqrt{3}}{2}mg$$
$R(\nwarrow)$:
$$mg\sin 30 - T - 0.4R_B = 0$$
$$T = \frac{1}{2}mg - \frac{4}{10}\times\frac{\sqrt{3}}{2}mg$$
$$T = mg\left(\frac{1}{2} - \frac{\sqrt{3}}{5}\right) \qquad \textbf{(2)}$$

$$\left(\frac{1}{2} - \frac{\sqrt{3}}{5}\right)m = 1 + \frac{\sqrt{3}}{5}$$
$$\left(5 - 2\sqrt{3}\right)m = 10 + 2\sqrt{3}$$
$$m = \frac{10 + 2\sqrt{3}}{5 - 2\sqrt{3}}$$

5 b Since $m = 10$ kg, $R_B = 5\sqrt{3}g$

$R(\nearrow)$ for A, using Newton's second law:

$2a = T - 2g\sin 30^\circ - 0.2R_A$

$2a = T - g - \frac{\sqrt{3}}{5}g$ **(1)**

$R(\searrow)$ for B, using Newton's second law:

$10a = 5g - 2\sqrt{3}g - T$ **(2)**

$5 \times \mathbf{(1)} = \mathbf{(2)} \Rightarrow$

$5T - 5g - \sqrt{3}g = 5g - 2\sqrt{3}mg - T$

$6T = 10g - \sqrt{3}g$

$T = \frac{5}{3}g - \frac{\sqrt{3}}{6}g$

Substituting this value into (**1**):

$2a = \left(\frac{5}{3}g - \frac{\sqrt{3}}{6}g\right) - g - \frac{\sqrt{3}}{5}g$

$2a = \frac{2}{3}g - \frac{11\sqrt{3}}{30}g$

$a = \left(\frac{1}{3} - \frac{11\sqrt{3}}{60}\right)g = 0.15474...$

The acceleration is 0.155 ms^{-2} (3s.f.).

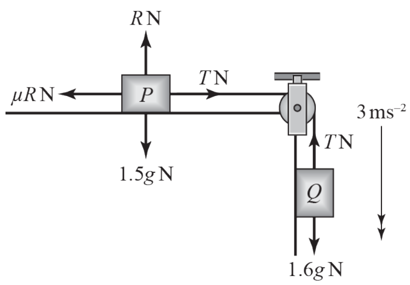
6 a $u = 0\ \text{ms}^{-1}$, $v = 6\ \text{ms}^{-1}$, $t = 2$ s, $a = ?$

$v = u + at$

$6 = 0 + 2a$

$a = \frac{6}{3} = 3$

The acceleration is 3 ms^{-2}



b Considering the box, Q, and using Newton's second law:

$F = ma$

$1.6g - T = 1.6 \times 3$

$T = 1.6g - 1.6 \times 3$

$T = 1.6 \times (9.8 - 3)$

$T = 10.88$

The tension in the string is 10.88 N.

6 c For P:

$R(\uparrow): R = 1.5g$

$R(\rightarrow)$:

$$F = ma$$
$$T - \mu R = ma$$
$$10.88 - 1.5\mu g = 1.5 \times 3$$
$$1.5\mu g = 10.88 - 4.5$$
$$\mu = \frac{6.38}{1.5 \times 9.8} = 0.43401...$$

To 3 s.f. the coefficient of friction is 0.434, as required.

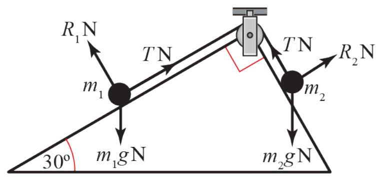
6 d The tension in the two parts of the string can be assumed to be the same because the string is inextensible.

Challenge

a With wedge smooth, let the reaction forces on the blocks be *R1* and *R2* respectively.
Resolving parallel to the slope on each side:

$T = m_1 g \sin 30^\circ$

$T = m_2 g \cos 30^\circ$



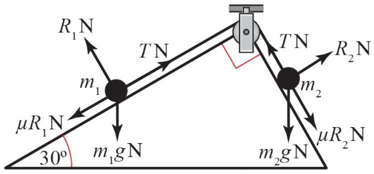
Since the string is inextensible, both values of T are the same, so:

$m_1 g \sin 30^\circ = m_2 g \cos 30^\circ$

$$\frac{m_1}{m_2} = \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{1}{\tan 30^\circ}$$

$$\frac{m_1}{m_2} = \sqrt{3} \text{ as required.}$$

Challenge



b Resolving perpendicular to the slope on each side:

$R_1 = m_1 g \cos 30^\circ$

$R_2 = m_2 g \sin 30^\circ$

<u>Case 1: m_1 is about to move down</u>

Resolving parallel to each slope to find tension in string if m_1 is about to move down:

$T = m_1 g \sin 30^\circ - \mu R_1 = m_1 g \sin 30^\circ - \mu m_1 g \cos 30^\circ$

$T = m_2 g \cos 30^\circ + \mu R_2 = m_2 g \cos 30^\circ + \mu m_2 g \sin 30^\circ$

Since the string is inextensible, both values of T are the same, so:

$$m_1 g\left(\sin 30^\circ - \mu \cos 30^\circ\right) = m_2 g\left(\cos 30^\circ + \mu \sin 30^\circ\right)$$

$$\frac{m_1}{m_2} = \frac{\cos 30^\circ + \mu \sin 30^\circ}{\sin 30^\circ - \mu \cos 30^\circ}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{3} + \mu}{1 - \mu\sqrt{3}}$$

<u>Case 1: m_2 is about to move down</u>

Resolving parallel to each slope to find tension in string if m_2 is about to move down:

$T = m_1 g \sin 30^\circ + \mu R_1 = m_1 g \sin 30^\circ + \mu m_1 g \cos 30^\circ$

$T = m_2 g \cos 30^\circ - \mu R_2 = m_2 g \cos 30^\circ - \mu m_2 g \sin 30^\circ$

Since the string is inextensible, both values of T are the same, so:

$$m_1 g\left(\sin 30^\circ + \mu \cos 30^\circ\right) = m_2 g\left(\cos 30^\circ - \mu \sin 30^\circ\right)$$

$$\frac{m_1}{m_2} = \frac{\cos 30^\circ - \mu \sin 30^\circ}{\sin 30^\circ + \mu \cos 30^\circ}$$

$$\frac{m_1}{m_2} = \frac{\sqrt{3} - \mu}{1 + \mu\sqrt{3}}$$

$\frac{m_1}{m_2}$ must lie between these two values, since they are the values of limiting equilibrium.

So:

$$\frac{\sqrt{3} - \mu}{1 + \mu\sqrt{3}} \le \frac{m_1}{m_2} \le \frac{\sqrt{3} + \mu}{1 - \mu\sqrt{3}} \quad \text{as required.}$$