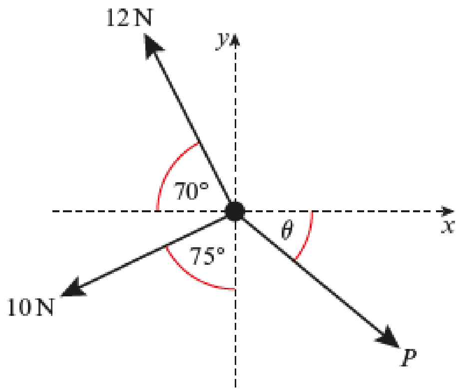
Applications of forces Mixed exercise 7

1 a Finding the components of P along each axis:



$R(\rightarrow): P_x = 12\cos 70^\circ + 10\sin 75^\circ$

$R(\uparrow): P_y = 12\sin 70^\circ - 10\cos 75^\circ$

$$\tan\theta = \frac{P_y}{P_x}$$

$$\tan\theta = \frac{12\sin 70^\circ - 10\cos 75^\circ}{12\cos 70^\circ + 10\sin 75^\circ} = 0.63124...$$

$\theta = 32.261...$

The angle θ is 32.3° (to 3s.f.).

b Using Pythagoras' theorem:

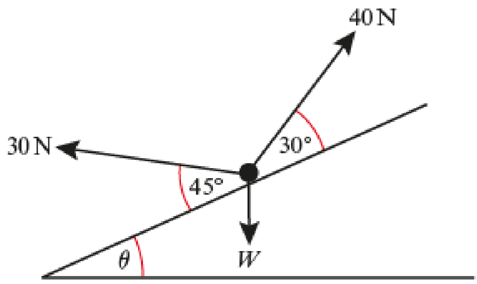
$$P^2 = P_x^{\ 2} + P_y^{\ 2}$$

$$P^2 = \left(12\cos 70^\circ + 10\sin 75^\circ\right)^2 + \left(12\sin 70^\circ - 10\cos 75^\circ\right)^2$$

$$P = \sqrt{264.91...} = 16.276...$$

P has a magnitude of 16.3 N (3s.f.).

2 a $R(\nwarrow)$:



$W\cos\theta = 40\sin 30^\circ + 30\sin 45^\circ$

$R(\swarrow)$:

$30\cos 45^\circ + W\sin\theta = 40\cos 30^\circ$

$W\sin\theta = 40\cos 30^\circ - 30\cos 45^\circ$

$$\frac{W\sin\theta}{W\cos\theta} = \frac{40\cos 30^\circ - 30\cos 45^\circ}{40\sin 30^\circ + 30\sin 45^\circ}$$

$$\tan\theta = \frac{20\sqrt{3} - 15\sqrt{2}}{20 + 15\sqrt{2}} = 0.35281...$$

$\theta = 18.046...$

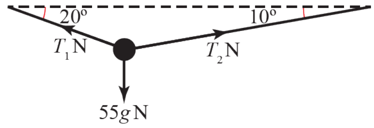
The angle θ is 18.0° (to 3s.f.).

b Using Pythagoras' theorem, $|W|^2$ is the sum of the squares of the two components.

$$W^2 = \left(20\sqrt{3} - 15\sqrt{2}\right)^2 + \left(20 + 15\sqrt{2}\right)^2$$

$$W = \sqrt{1878.8...} = 43.345...$$

The weight of the particle is 43.3 N (3s.f.).



3 Resolving horizontally:

$$T_1 \cos 20^\circ = T_2 \cos 10^\circ$$

$$T_2 = \frac{T_1 \cos 20^\circ}{\cos 10^\circ} \qquad \textbf{(1)}$$

Resolving vertically:

$$T_1 \sin 20^\circ + T_2 \sin 10^\circ = 55g \qquad \textbf{(2)}$$

Substituting $T_2 = \dfrac{T_1 \cos 20^\circ}{\cos 10^\circ}$ from (**1**) into (**2**):

$$T_1 \sin 20^\circ + \frac{T_1 \cos 20^\circ \times \sin 10^\circ}{\cos 10^\circ} = 55g$$

$$T_1(\sin 20^\circ + \cos 20^\circ \tan 10^\circ) = 55g$$

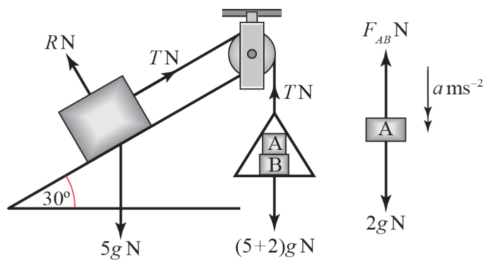
$$T_1 = \frac{55 \times 9.8}{\sin 20^\circ + \cos 20^\circ \tan 10^\circ} = 1061.6...$$

Subsituting this value of T_1 into (**1**):

$$T_2 = \frac{\cos 20^\circ \times 1061.6...}{\cos 10^\circ} = 1012.9...$$

The tensions in the two parts of the rope are 1062 N and 1013 N (nearest whole number).

4 Let the acceleration of the system be a ms^{-2}



For the block, $R(\nearrow)$:

$$F = ma$$

$$T - 5g \sin 30^\circ = 5a$$

$$T = 5a + \frac{5g}{2} \qquad \textbf{(1)}$$

For the pan and masses, $R(\downarrow)$:

$$F = ma$$

$$(2+5)g - T = (2+5)a$$

$$T = 7g - 7a \qquad \textbf{(2)}$$

Since the string is inextensible, T is constant and hence (**1**) = (**2**):

$$5a + \frac{5g}{2} = 7g - 7a$$

$$12a = \frac{9g}{2}$$

$$a = \frac{3g}{8}$$

Hence, the scale-pan accelerates downward, away from the pulley with magnitude $a = \dfrac{3g}{8}$ ms^{-2}

Applying Newton's second law to mass A, and denoting the force exerted by B on A as F_{AB}:

$$2g - F_{AB} = 2a$$

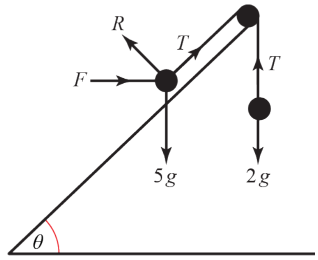
$$F_{AB} = 2g - \frac{2 \times 3g}{8}$$

$$F_{AB} = \frac{10}{8}g$$

$$F_{AB} = 1.25 \times 9.8 = 12.25$$

4 However, Newton's third law of motion gives
|force exerted by B on A| = |force exerted by A on B|.
Therefore the force exerted by A on B is 12.25 N.

5 a For the 2 kg particle



$R(\uparrow)$

$T - 2g = 0$

$\therefore T = 2g$

For the 5 kg particle:

$R(\nearrow)$

$T + F\cos\theta - 5g\sin\theta = 0$

$\therefore F\cos\theta = 5g\sin\theta - T$

As $T = 2g$, $\cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{3}{5}$

$F \times \frac{4}{5} = 5g \times \frac{3}{5} - 2g$

$\therefore F = g \times \frac{5}{4}$

$= \frac{5g}{4}$

$= 12$ (2 s.f.)

b $R(\nwarrow)$

$R - F\sin\theta - 5g\cos\theta = 0$

$\therefore R = F\sin\theta + 5g\cos\theta$

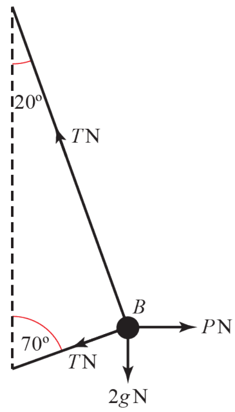
$= \left(\frac{5}{4}g \times \frac{3}{5}\right) + \left(5g \times \frac{4}{5}\right)$

$= \frac{19}{4}g$

$= 47$ (2 s.f.)

c F will be smaller

6 a Resolving vertically:



$T\cos 20^\circ = T\cos 70^\circ + 2g$

$T(\cos 20^\circ - \cos 70^\circ) = 2g$

$T = \frac{2 \times 9.8}{\cos 20^\circ - \cos 70^\circ} = 32.793...$

The tension in the string, to two significant figures, is 33 N.

6 b Resolving horizontally:

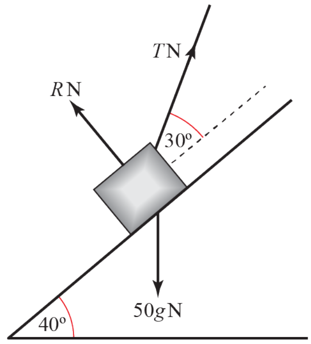
$P = T\sin 20^\circ + T\sin 70^\circ$

$P = (\sin 20^\circ + \sin 70^\circ) \times 32.793...$

$P = 42.032...$

The value of P is 42 N (2 s.f.).

7 a $R(\nearrow)$:



$T\cos 30^\circ = 50g\sin 40^\circ$

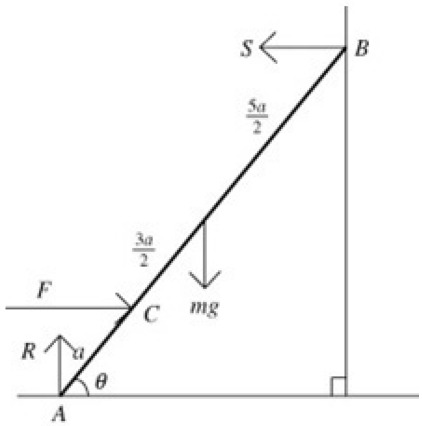
$$T = \frac{50 \times 9.8\sin 40^\circ}{\cos 30^\circ}$$

$$= 363.69...$$

The tension in the string is 364 N (3s.f.).

b Even when the hill is covered in snow, there is likely to be some friction between the runners of the sled and the slope, so modelling the hill as a smooth slope is unrealistic.

8 Let S be the reaction of the wall on the ladder at B.
Let R be the reaction of the ground on the ladder at A.
(Both surfaces are smooth, so no friction.)



$R(\rightarrow)$: $F = S$

Taking moments about A:

$$mg \times \frac{5}{2}a \times \cos\theta + F \times a \times \sin\theta = S \times 5a \times \sin\theta$$

$$\frac{5mg}{2} + F\tan\theta = 5S\tan\theta \qquad \text{(dividing by } a\cos\theta)$$

$$\frac{5mg}{2} + F\tan\theta = 5F\tan\theta \qquad \text{(Since } F = S)$$

$$\frac{5mg}{2} = 4F\tan\theta$$

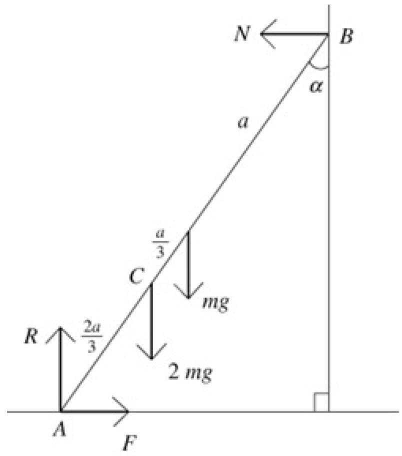
$$= 4 \times \frac{9}{5}F \qquad \text{(Since } \tan\theta = \frac{9}{5})$$

$$= 7.2F$$

$$F = \frac{5mg}{2 \times 7.2}$$

$$= \frac{25mg}{72} \text{ as required.}$$

9 Let N be the reaction of the wall on the ladder at B.
Let R be the reaction of the ground on the ladder at A,
Let F the friction between the ladder and the ground at A.



$$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5} \text{ and } \cos\alpha = \frac{4}{5}$$

$$R(\uparrow):\quad R = mg + 2mg = 3mg$$

Taking moments about B:

$$mg \times a\sin\alpha + 2mg \times \frac{4}{3}a\sin\alpha + F \times 2a\cos\alpha = R \times 2a\sin\alpha$$

$$mga \times \frac{3}{5} + \frac{8mga}{3} \times \frac{3}{5} + F \times 2a \times \frac{4}{5} = 6mga \times \frac{3}{5}$$

$$F \times \frac{8a}{5} = \frac{18mga}{5} - \frac{8mga}{5} - \frac{3mga}{5}$$

$$F \times \frac{8a}{5} = \frac{7mga}{5}$$

$$F = \frac{7mga}{5} \times \frac{5}{8a}$$

$$= \frac{7mg}{8}$$

The ladder and the child are in equilibrium, so

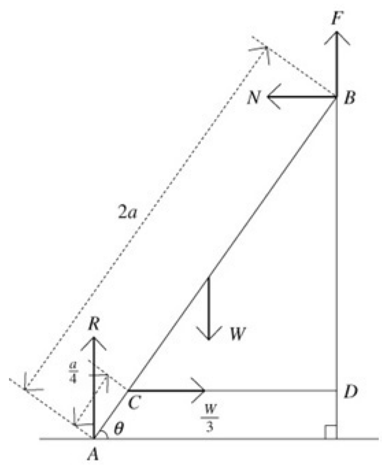
$$F \le \mu R$$

$$\frac{7mg}{8} \le \mu \times 3mg$$

$$\mu \ge \frac{7}{24}$$

The least possible value for μ is $\frac{7}{24}$

10 Let R be the reaction of the ground on the ladder at A,
Let N be the reaction of the wall on the ladder at B
Let F be the friction between the wall and the ladder at B.



a Since you do not know the magnitude of F, you cannot resolve vertically to find R.
Therefore, take moments about B (since this eliminates F):

$$\frac{W}{3} \times \frac{7a}{4}\sin\theta + W \times a\cos\theta = R \times 2a\cos\theta$$

$$\frac{7W}{12} \times \tan\theta + W = 2R \qquad \text{(dividing through by } a\cos\theta\text{)}$$

$$\frac{7}{12} \times \frac{4}{3}W + W = 2R \qquad \text{(since } \tan\theta = \frac{4}{3}\text{)}$$

$$\frac{16W}{9} = 2R$$

$$R = \frac{8W}{9}$$

10 b $R(\rightarrow)$: $\quad N = \dfrac{W}{3}$

$R(\uparrow)$:

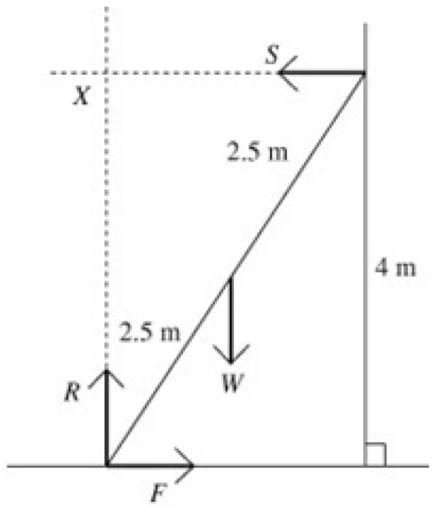
$$R + F = W$$
$$F = W - R$$
$$= W - \frac{8}{9}W$$
$$= \frac{W}{9}$$

For the ladder to remain in equilibrium,

$$F \le \mu N$$
$$\frac{W}{9} \le \mu \frac{W}{3}$$
$$\mu \ge \frac{1}{3}$$

c The ladder had negligible thickness / the ladder does not bend.

11 a Let S be the reaction of the wall on the ladder
Let R be the reaction of the ground on the ladder
Let F the friction between the ladder and the ground
Let X be the point where the lines of action of R and S intersect, as shown in the diagram.



By Pythagoras's Theorem, distance from base of ladder to wall is 3 m.

$R(\rightarrow)$: $\quad F = S$

$R(\uparrow)$: $\quad R = W$

Taking moments about X:
$1.5W = 4F$

Suppose the ladder can rest in equilibrium in this position. Then

$$F \le \mu R$$
$$\frac{1.5W}{4} \le 0.3 \times W$$
$$\frac{3W}{8} \le \frac{3W}{10}$$
$$30 \le 24$$

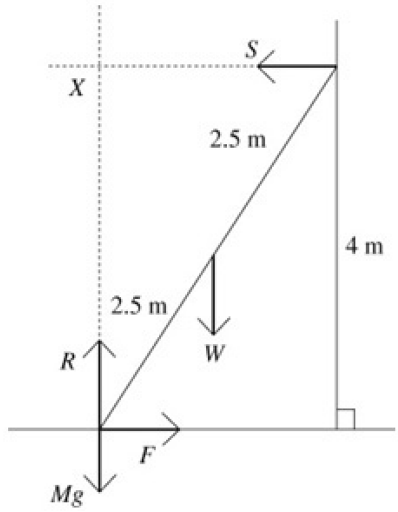
which is false, therefore the assumption that $F \le \mu R$ must be false – the ladder cannot be resting in equilibrium.

11 b With the brick in place, take moments about X:

$1.5W = 4F$ so

$$F = \frac{1.5W}{4} = \frac{3W}{8}$$

which is independent of M, the mass of the brick.



c $R(\uparrow)$ $R = W + Mg$

$R(\rightarrow)$ $F = S$

$$F \leq \mu R \Rightarrow \frac{3W}{8} \leq 0.3(W + Mg) = \frac{3(W + Mg)}{10}$$

$$\Rightarrow 10W \leq 8W + 8Mg$$

$$8Mg \geq 2W, \ M \geq \frac{W}{4g}$$

So the smallest value for M is $\frac{W}{4g}$

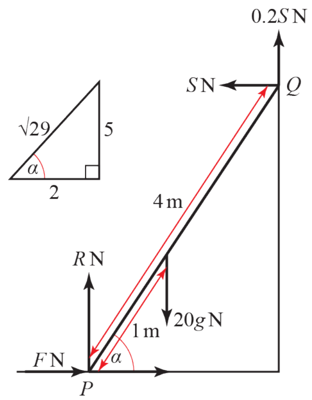
12 Let S be the reaction of the wall on the ladder at Q

Let R be the reaction of the ground on the ladder at P

$\tan\alpha = \frac{5}{2} \Rightarrow \sin\alpha = \frac{5}{\sqrt{29}}$ and $\cos\alpha = \frac{2}{\sqrt{29}}$

Since the ladder is in limiting equilibrium,

frictional force at the wall $= \mu S = 0.2S$.



Taking moments about P:

$$20g \times 1\cos\alpha = S \times 4\sin\alpha + 0.2S \times 4 \times \cos\alpha$$

$$\frac{20 \times 2}{\sqrt{29}}g = \left(\frac{4 \times 5}{\sqrt{29}} + \frac{0.8 \times 2}{\sqrt{29}}\right)S$$

$$40g = 21.6S$$

$$S = \frac{392}{21.6} = 18.148...$$

R($\rightarrow$): $F = S$

The force F required to hold the ladder still is 18 N (2 s.f.).

13 Since the rod is uniform, the weight acts from the midpoint of AB.

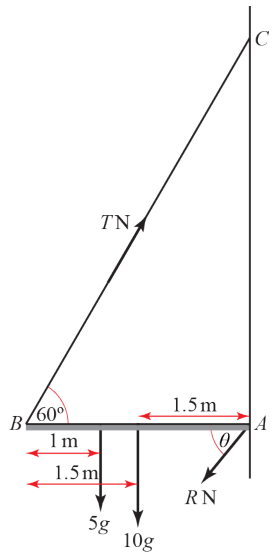
a Take moments about A:

$$(10g \times 1.5) + (5g \times 2) = T \times 3\sin 60^\circ$$

$$T = \frac{25g}{3\sin 60^\circ}$$

$$T = \frac{25 \times 9.8 \times 2}{3\sqrt{3}} = 94.300...$$

The tension in the string is 94.3 N (3s.f.).



b Let the horizontal and vertical components of the reaction R at the hinge be R_x and R_y respectively.

Resolving horizontally:

$$R_x = T\cos 60^\circ$$
$$R_x = \frac{25g}{3\sin 60^\circ}\cos 60^\circ$$
$$R_x = \frac{25g}{3\tan 60^\circ} = \frac{25g}{3\sqrt{3}}$$

Resolving vertically upwards:

$$R_y = T\sin 60^\circ - 10g - 5g$$
$$R_y = \frac{25g}{3\sin 60^\circ}\sin 60^\circ - 15g$$
$$R_y = \left(\frac{25}{3} - 15\right)g = -\frac{20g}{3}$$

The reaction at the hinge is given by:

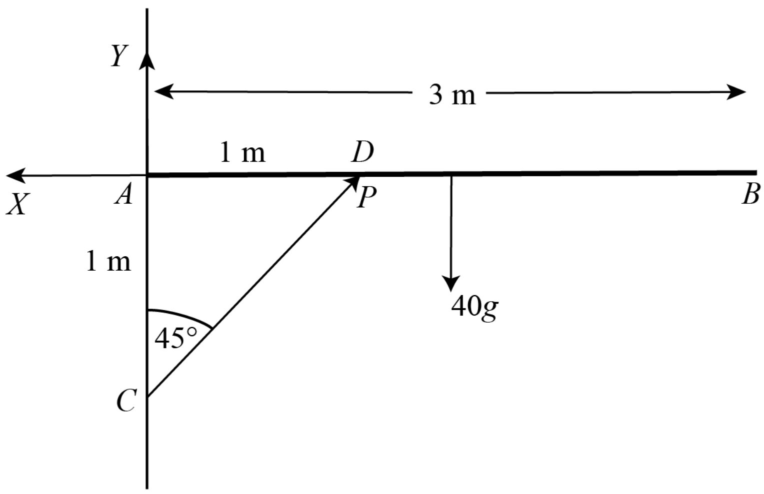
$$R^2 = R_x^{\ 2} + R_y^{\ 2}$$
$$R^2 = \left(\frac{25g}{3\sqrt{3}}\right)^2 + \left(-\frac{20g}{3}\right)^2$$
$$R^2 = \left(\frac{625}{27} + \frac{400}{9}\right)g^2$$
$$R = \sqrt{\frac{1825}{27}} \times 9.8 = 80.570...$$
$$\tan\theta = \frac{R_y}{R_x}$$
$$\tan\theta = \frac{20g}{3} \times \frac{3\sqrt{3}}{25g} = \frac{4\sqrt{3}}{5}$$
$$\theta = 54.182...$$

The reaction at the hinge is 80.6 N acting at 54.2° below the horizontal (both values to 3 s.f.).

14 Let the horizontal and vertical components or the force at A be X and Y respectively. Let the thrust in the rod be P.

a M(A): $1 \times P \times \cos 45° = 40g \times \frac{3}{2}$

$P = \frac{60g}{\cos 45°} = 60\sqrt{2}g = 830$ N (2 s.f.)



b $\text{R}(\rightarrow): X = P\cos 45° = 60g$

$\text{R}(\uparrow): Y + P\cos 45° = 40g$

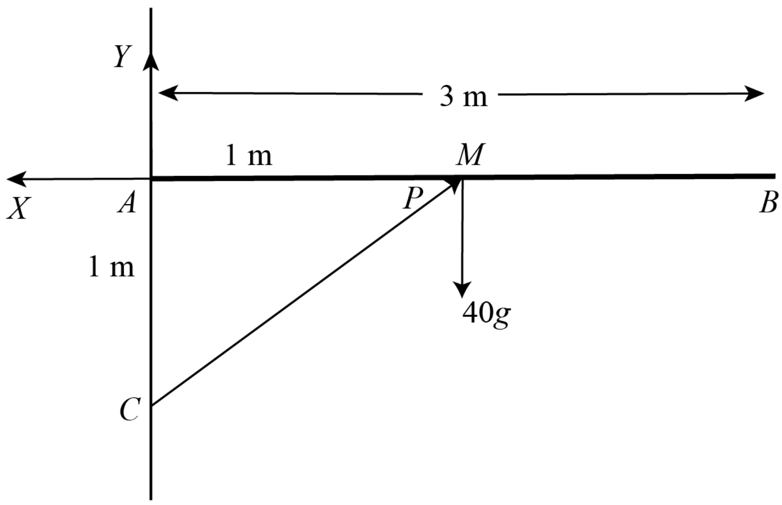
$Y = 40g - 60g = -20g$

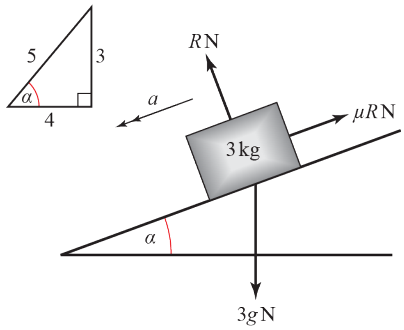
resultant $= \sqrt{X^2 + Y^2}$

$= 10g\sqrt{4^2 + 2^2} = 10g\sqrt{40}$

$= 620$ N (2 s.f.)

c The lines of action of P and the weight meet at M, hence the line of action of the resultant of X and Y must also pass through M (3 forces acting on a body in equilibrium). Therefore the reaction must act horizontally (i.e. no vertical component).





15 $\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

$u = 0\text{ ms}^{-1}$, $s = 6$ m, $t = 1.5$ s, $a = ?$

$$s = ut + \frac{1}{2}at^2$$

$$6 = 0 + \frac{1}{2}a \times 1.5^2 = \frac{9a}{8}$$

$$a = 6 \times \frac{9}{8} = \frac{16}{3}$$

$R(\nwarrow)$: $R = 3g\cos\alpha$

$R(\swarrow)$:

$$F = ma$$

$$3g\sin\alpha - \mu R = 3 \times \frac{16}{3}$$

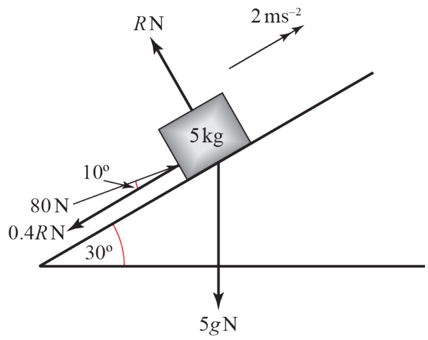
$$3g\sin\alpha - (\mu \times 3g\cos\alpha) = 16$$

$$\frac{3}{5}g - \frac{4}{5}\mu g = \frac{16}{3}$$

$$9g - 12\mu g = 80$$

$$\mu = \frac{(9 \times 9.8) - 80}{12 \times 9.8} = 0.06972...$$

The coefficient of friction is 0.070 (3s.f.).



16 $R(\nwarrow)$: $R = 5g\cos 30^\circ + 80\sin 10^\circ$

$R(\nearrow)$:

$ma = F$

$$5a = 80\cos 10^\circ - 0.4R - 5g\sin 30^\circ$$

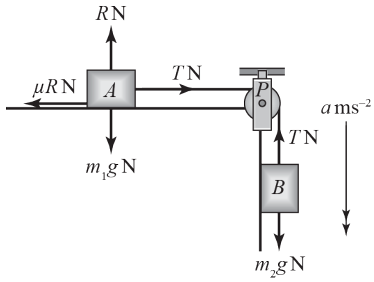
$$5a = 80\cos 10^\circ - 0.4\left(5g\cos 30^\circ + 80\sin 10^\circ\right) - 5g\sin 30^\circ$$

$$a = 16\cos 10^\circ - 0.4\left(g\cos 30^\circ + 16\sin 10^\circ\right) - g\sin 30^\circ$$

$$a = 6.3507...$$

The acceleration of the particle is 6.53 ms^{-2} (3s.f.).

17 Since $m_2 > \mu m_1$, when system is released from rest then B moves downwards and A moves towards the pulley P



For particle A:

$R(\uparrow)$: $R = \mu m_1 g$

$R(\rightarrow)$:

$$F = m_1 a$$
$$T - \mu R = m_1 a$$
$$T - \mu m_1 g = m_1 a$$
$$T = m_1 a + \mu m_1 g \qquad \textbf{(1)}$$

For particle B:

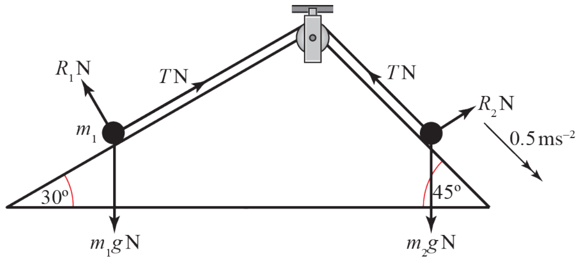
$R(\uparrow)$:

$$F = m_2 a$$
$$m_2 g - T = m_2 a$$
$$T = m_2 g - m_2 a \qquad \textbf{(2)}$$

Since string is inextensible, the values of T are identical, so (**1**) = (**2**):

$$m_1 a + \mu m_1 g = m_2 g - m_2 a$$
$$m_1 a + m_2 a = m_2 g - \mu m_1 g$$
$$a = \frac{g(m_2 - \mu m_1)}{m_1 + m_2} \quad \text{as required.}$$

18 For particle with mass m_1:



$R(\nearrow)$

$$F = ma$$

$$T - m_1 g \sin 30^\circ = \frac{m_1}{2}$$

$$T = \left(\frac{1}{2} + g \sin 30^\circ\right) m_1 \qquad \textbf{(1)}$$

For particle with mass m_2:

$R(\nearrow)$

$$F = ma$$

$$m_2 g \sin 45^\circ - T = \frac{m_2}{2}$$

$$\left(g \sin 45^\circ - \frac{1}{2}\right) m_2 = T \qquad \textbf{(2)}$$

Since string is inextensible, T is constant throughout and hence (**1**) = (**2**):

$$\left(\frac{1}{2} + g \sin 30^\circ\right) m_1 = \left(g \sin 45^\circ - \frac{1}{2}\right) m_2$$

$$\left(\frac{1}{2} + \frac{g}{2}\right) m_1 = \left(\frac{g\sqrt{2}}{2} - \frac{1}{2}\right) m_2$$

$$\frac{m_1}{m_2} = \frac{g\sqrt{2} - 1}{1 + g} \quad \text{as required}$$